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Modeling Carbon Fiber Failure Times Using the New Exponentiated Inverse Rayleigh (NEIR) Distribution

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ABSTRACT

In this paper, we propose and explore the New Exponentiated Inverse Rayleigh (NEIR) distribution. The NEIR distribution enhances modeling flexibility for right-skewed lifetime data. We derive its key statistical properties including the survival function, hazard function, quantile function, moments, moment generating function, characteristic function, mode and order statistics. A simulation algorithm is also described to access the performance of the estimates. The model parameters estimated using maximum likelihood estimation method and its practical application is discussed to highlight the model's flexibility and adaptability using a real dataset.

Keywords: NEIR, Statistical Properties, Renyi Entropy, Mean Residual Life, Maximum Likelihood function

1. Introduction

In statistics there are large number of continuous distributions was introduced in the literary work for modeling data in different fields, statistics have important role in different fields like Business and economics, science and research, social sciences, sports analytics, engineering and many other fields. Most of the researchers have wide contributions in exponential inverse Rayleigh inverse distribution such as Banerjee et.al [1] introduce Exponential transformed inverse Rayleigh distribution, Hussain et.al [2] introduce discrete inverse Rayleigh distribution Mohammed et.al [3] introduce The inverse exponential Rayleigh distribution, Kamnge et.al [4] Introduce Half logistic exponentiated inverse Rayleigh distribution, Dey [5] introduce Bayesian estimation of the parameter and reliability function of an inverse Rayleigh distribution, Ali et.al [6] introduce Alpha-Power Exponentiated Inverse Rayleigh distribution, ,Khanet.al [7] introduce Transmuted modified inverse Rayleigh distribution, Soliman et.al [8] introduce Estimation and prediction from inverse Rayleigh distribution based on lower record values, Chiodo et.al [9] introduce Stochastic extreme wind speed modeling and bayes estimation under the inverse Rayleigh distribution, Rao et.al [10] introduce Estimation of stress–strength reliability from exponentiated inverse Rayleigh distribution.

The aim of this article is to introduce a new probability model the new model is called New Exponentiated Inverse Rayleigh distribution and to study its statistical properties. To estimate the parameter of the new model using maximum likelihood estimation method. To access goodness-of fit of the new model by using a real dataset.

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2. The Derived Distribution

Let G(x) and g(x) be the cumulative distribution function (cdf) and probability density function (pdf) of any random variable x. Then the cdf and pdf of new exponentiated family class introduce by Anwar Hassan *et.al* [2] is given by

$$G(x) = 2^{F(x)} - 1;\ x \in \mathbb{R} \tag{1}$$

and the corresponding pdf

$$g(x) = \log(2) f(x) 2^{F(x)};\ x \in \mathbb{R} \tag{2}$$

Let X be a random variable with exponentiated inverse Rayleigh distribution having cumulative distribution $G(x) = \exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma};\ x, \theta, \gamma > 0$ then the cdf of new exponentiated inverse Rayleigh distribution (NEIR) is given by

$$G(x) = 2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}} - 1 \tag{3}$$

and the corresponding pdf

$$g(x) = \frac{\gamma\theta \log(2) 2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]} \exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3} \tag{4}$$

The survival function (sf) and hazard function (hf) is given by

$$S(x) = 2 - 2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}} \tag{5}$$

and

$$H(x) = \frac{\gamma\theta \log(2) 2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]} \exp\left[-\frac{\gamma\theta}{x^2}\right]}{\left[2 - 2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}}\right] x^3} \tag{6}$$

(a) (b)

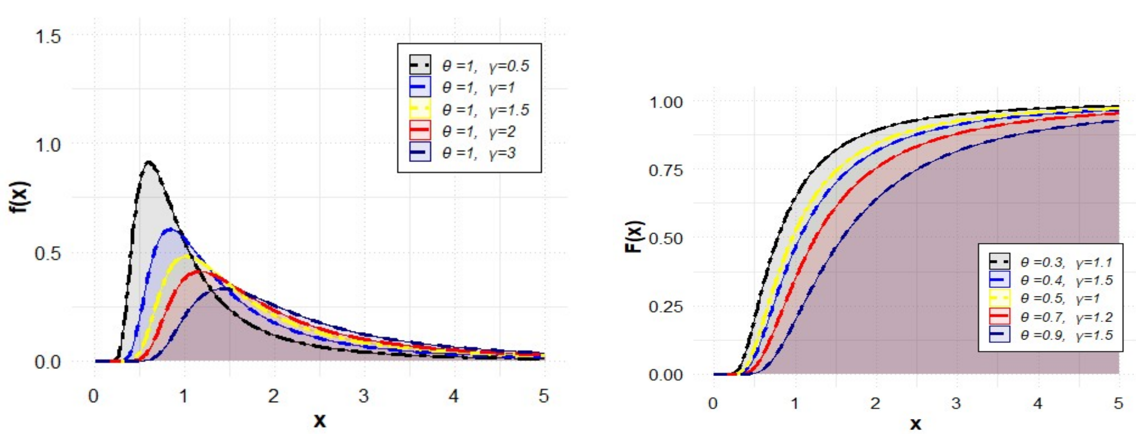

Figure 1: Plots for the pdf and cdf for the selected values of the parameters

(a) (b)

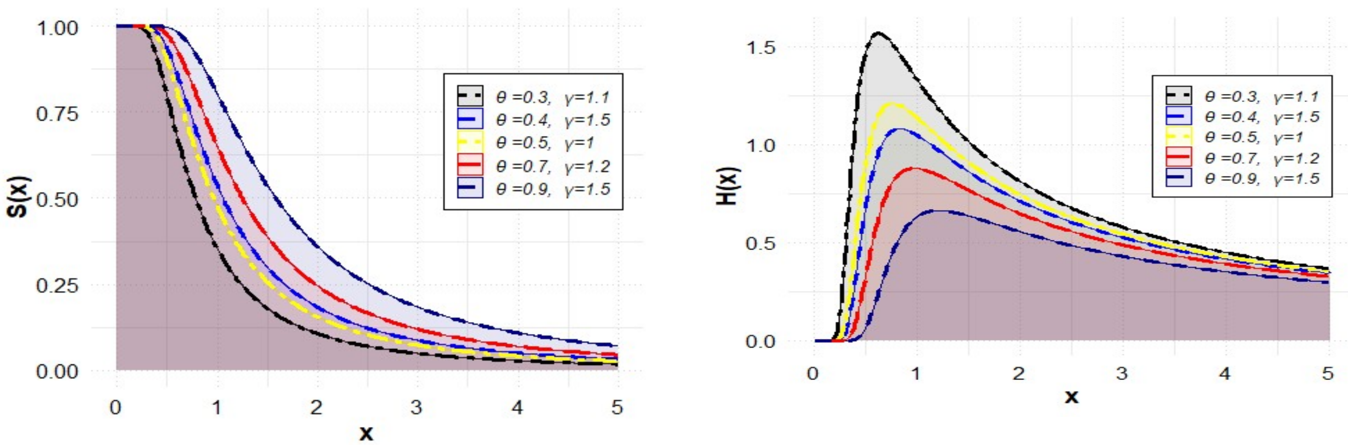

Figure 2: Plots for the S(x) and h(x) for the selected values of the parameters

Table 1: Moments of the NEIR model

θ	γ	Mean	Standard Deviation	Skewness	Kurtosis
0.25	0.20	0.47713	0.93055	170.204	92074.88
	0.30	0.5701692	1.124496	144.4044	64717.7
	0.40	0.6458262	1.28639	128.4081	50351.51
	0.50	0.7106428	1.426439	117.566	41605.17
	1.00	0.9515099	1.966229	89.32557	22997.32
0.50	0.20	0.6458262	1.28639	128.4081	50351.51
	0.30	0.7679166	1.552449	109.3081	35566.75
	0.40	0.8668697	1.773726	97.518	27804.01
	0.50	0.9515099	1.966229	89.32557	22997.32
	1.00	1.266025	2.701744	68.39356	12861.57
1.00	0.20	0.8668697	1.773726	97.518	27804.01
	0.30	1.026255	2.138412	83.19423	19711.23
	0.40	1.155416	2.441054	74.36267	15457.79
	0.50	1.266025	2.701744	68.39356	12861.57
	1.00	1.679325	3.695736	52.93758	7313.1
1.25	0.20	0.9515099	1.966229	89.32557	22997.32
	0.30	1.125177	2.369014	76.32165	16341.45
	0.40	1.266025	2.701744	68.39356	12861.57
	0.50	1.386769	2.990106	62.89612	10702.23
	1.00	1.839257	4.083082	48.88162	6123.908

3. Statistical Properties

In this section, we derive some statistical properties of the NEIR model including quantile function, *rth* moment formula, moment generating function, characteristics function and model.

3.1. Quantile function

The quantile function of the NEIR model is

$$u = G(x)$$

$$x = G^{-1}(u)$$

$$u = 2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}} - 1$$

Taking natural log on both sides

$$\log(1+u) = \exp\left[-\frac{\theta\gamma}{x^2}\right]\log(2)$$

$$\frac{\log(1+u)}{\log(2)} = \exp\left[-\frac{\theta\gamma}{x^2}\right]$$

Solving for x we get

$$x = \sqrt{\frac{-\theta\gamma}{\log\left[\frac{\log(u+1)}{\log(2)}\right]}} \tag{7}$$

The median of the NEIR model can be obtained by putting $u = \frac{1}{2}$

$$x = \sqrt{\frac{-\theta\gamma}{\log\left[\frac{\log\left(\frac{1}{2}+1\right)}{\log(2)}\right]}} \tag{8}$$

3.2. rth Moment

By definition the *rth* moment is given by

$$\mu_r' = \int_0^\infty x^r g(x)\,dx$$

$$\mu_r' = \int_0^\infty x^r \frac{\gamma\theta\log(2)\,2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]}\exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3}dx$$

Let

$$u = \frac{\gamma\theta}{x^2} \quad -du = \frac{2\gamma\theta}{x^3}dx$$

$$x = \left[\frac{\gamma\theta}{u}\right]^{\frac{r}{2}}$$

$$if\ x = 0 \quad then \quad u = \infty$$

$$if\ x = \infty \quad then \quad u = 0$$

$$\mu_r' = \log(2)\int_{\infty}^{0}\left(\frac{\gamma\theta}{u}\right)^{\frac{r}{2}} 2^{\exp[-u]}\exp[-u] - du$$

$$\mu_r' = \log(2)(\gamma\theta)^{\frac{r}{2}}\int_{0}^{\infty} u^{-\frac{r}{2}} 2^{\exp[-u]}\exp[-u]\,du$$

Now

$$t = \exp(-u) \quad -dt = \exp(-u)\,du \quad u = -\log(t)$$
$$if\ u = 0 \quad then \quad t = 1$$
$$if\ u = \infty \quad then \quad t = 0$$

$$\mu_r' = \log(2)(\gamma\theta)^{\frac{r}{2}}\int_{1}^{0}(-\log(t))^{\frac{r}{2}} 2^{t} - dt$$

$$\mu_r' = \log(2)(\gamma\theta)^{\frac{r}{2}}\int_{0}^{1}(-\log(t))^{\frac{r}{2}} 2^{t}\,dt$$

Using power series

$$2^{t} = \sum_{n=0}^{\infty}\frac{(\log(2))^{n}}{n!}t^{n}$$

$$\mu_r' = \sum_{n=0}^{\infty}\frac{(\log(2))^{n+1}}{n!}(\gamma\theta)^{\frac{r}{2}}\int_{0}^{1}(-\log(t))^{\frac{r}{2}} t^{n}\,dt$$

$$\mu_r' = \sum_{n=0}^{\infty}\frac{(\log(2))^{n+1}}{n!}(\gamma\theta)^{\frac{r}{2}}\frac{\Gamma\left(-\frac{r}{2}+1\right)}{(n+1)^{-\frac{r}{2}+1}} \tag{9}$$

3.3. Moment Generating Function

By definition moment generating function is given by

$$M_x(t) = E(\exp(tx))$$

$$M_x(t) = \int_{0}^{\infty}\exp(tx)\,g(x)\,dx$$

Using Taylors expansion

$$M_x(t) = \sum_{r=0}^{\infty}\frac{t^{r}}{r!}\int_{0}^{\infty}x^{r} g(x)\,dx$$

$$M_x(t) = \sum_{r=0}^{\infty}\frac{t^{r}}{r!}\mu_r'$$

$$M_x(t) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(\log(2))^{n+1}}{n!}\frac{t^{r}}{r!}(\gamma\theta)^{\frac{r}{2}}\frac{\Gamma\left(-\frac{r}{2}+1\right)}{(n+1)^{-\frac{r}{2}+1}} \tag{10}$$

3.4. Characteristics Function

By definition characteristics is given by

$$\phi_x(t) = E(\exp(itx))$$

$$\phi_x(t) = \int_0^\infty \exp(itx)\, g(x)\, dx$$

Using Taylors expansion

$$\phi_x(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \int_0^\infty x^r g(x)\, dx$$

$$\phi_x(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \mu_r'$$

$$\phi_x(t) = \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \frac{(\log(2))^{n+1}}{n!} \frac{(it)^r}{r!} (\gamma\theta)^{\frac{r}{2}} \frac{\Gamma\left(-\frac{r}{2}+1\right)}{(n+1)^{-\frac{r}{2}+1}} \tag{11}$$

3.5 Mode

By definition mode is

$$\frac{d}{dx} g(x) = 0$$

$$\frac{2^{1+\exp\left(-\frac{\theta\gamma}{x^2}\right)} \gamma\theta \log(2) \left(-3x^2 + \gamma\theta\left(2 + \exp\left(-\frac{\theta\gamma}{x^2}\right) \log[4]\right)\right)}{x^6} = 0 \tag{12}$$

The equation (12) has not exacted form so one can use iterative procedures to get numerical solution.

4. Renyi Entropy

By definition

$$H(\beta) = \frac{1}{1-\beta} \log\left[\int_0^\infty \{g(x)\}^\beta\, dx\right]$$

$$H(\beta) = \frac{1}{1-\beta} \log\left[\int_0^\infty \left\{ \frac{\gamma\theta \log(2)\, 2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]} \exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3} \right\}^\beta dx\right]$$

$$H(\beta) = \frac{1}{1-\beta} \log\left[\gamma\theta 2 \log(2)\right]^\beta \int_0^\infty \frac{2^{\exp\left(-\frac{\gamma\theta\beta}{x^2}\right)} \exp\left(-\frac{\gamma\theta\beta}{x^2}\right)}{x^{3\beta}} dx$$

Considering

$$\int_0^\infty \frac{2^{\exp\left(-\frac{\gamma\theta\beta}{x^2}\right)}\exp\left(-\frac{\gamma\theta\beta}{x^2}\right)}{x^{3\beta}}dx$$

Let

$$u=\frac{\gamma\theta\beta}{x^2} \quad -du=\frac{2\gamma\theta\beta}{x^3}dx \quad dx=\frac{-x^3dx}{2\gamma\theta\beta}$$

$$\frac{u}{\gamma\theta\beta}=\frac{1}{x^2} \quad x=\left[\frac{\gamma\theta\beta}{u}\right]^{1/2}$$

$$=\int_0^\infty \frac{2^{\exp(-u)}\exp(-u)\left(\frac{\gamma\theta\beta}{u}\right)^{3/2}\frac{du}{2\gamma\theta\beta}}{\left(\frac{\gamma\theta\beta}{u}\right)^{3\beta/2}}$$

After simplification the integral reduces to

$$=\frac{(\gamma\theta\beta)^{3\beta/2}}{2}\int_0^\infty u^{3/2(\beta-1)}2^{\exp(-u)}\exp(-u)\,du$$

Now

$$t=\exp(-u) \quad -dt=\exp(-u)\,du \quad u=-\log t$$

$$f\ u=0 \quad then \quad t=1$$

$$if\ u=\infty \quad then \quad t=0$$

$$=\frac{(\gamma\theta\beta)^{3\beta/2}}{2}\int_0^1(-\log t)^{3/2(\beta-1)}2^t\,dt$$

Using power series

$$2^t=\sum_{n=0}^\infty\frac{(\log(2))^n}{n!}t^n$$

$$=\sum_{n=0}^\infty\frac{(\log 2)^n}{n!}\frac{(\gamma\theta\beta)^{3\beta/2}}{2}\int_0^1(-\log t)^{3/2(\beta-1)}t^n\,dt$$

$$=\sum_{n=0}^\infty\frac{(\log 2)^n}{n!}\frac{(\gamma\theta\beta)^{3\beta/2}}{2}\frac{\Gamma\left[3\beta/2-1/2\right]}{(n+1)^{3\beta/2-1/2}}$$

$$H(\beta)=\frac{1}{1-\beta}\log\sum_{n=0}^\infty\frac{(\log 2)^{n+\beta}}{n!}(\gamma\theta)^{5\beta/2}\beta^{3\beta/2}2^{\beta-1}\frac{\Gamma\left[3\beta/2-1/2\right]}{(n+1)^{3\beta/2-1/2}} \tag{13}$$

5. Mean Residual Life

By definition

$$\mu(t)=\frac{1}{S(t)}\int_t^\infty xg(x)\,dx-t$$

$$\mu(t)=\frac{1}{S(t)}\int_t^\infty x\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]}\exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3}dx-t$$

Considering

$$\int_t^\infty x\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]}\exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3}dx$$

Let

$$u=\frac{\gamma\theta}{x^2}\quad -du=\frac{2\gamma\theta}{x^3}dx\quad x=\left(\frac{\gamma\theta}{u}\right)^{1/2}$$

$$if\ x=t\quad then\quad u=\frac{\gamma\theta}{t^2}$$

$$if\ x=\infty\quad then\quad u=0$$

$$=\log(2)(\gamma\theta)^{1/2}\int_0^{\gamma\theta/t^2}u^{-1/2}\exp(-u)2^{\exp(-u)}du$$

Now

$$t=\exp(-u)\quad -dt=\exp(-u)du\quad u=-\log(t)$$

$$if\ u=0\quad then\quad t=1$$

$$if\ u=\frac{\gamma\theta}{t^2}\quad then\quad t=\exp\left(\frac{\gamma\theta}{t^2}\right)$$

$$=\log(2)(\gamma\theta)^{1/2}\int_1^{\exp\left(\frac{\gamma\theta}{t^2}\right)}\left(-\log(t)\right)^{-1/2}2^t\,dt$$

Using power series

$$2^{t}=\sum_{n=0}^{\infty}\frac{\left(\log\left(2\right)\right)^{n}}{n!}t^{n}$$

$$=\left[\log\left(2\right)\right]^{n+1}\left(\gamma\theta\right)^{1/2}\int_{1}^{\exp\left(\frac{\gamma\theta}{t^{2}}\right)}\left(-\log\left(t\right)\right)^{-1/2}tdt$$

$$=\left[\log\left(2\right)\right]^{n+1}\left(\gamma\theta\right)^{1/2}\left[\sqrt{\pi}-\frac{t}{\sqrt{2\gamma\theta}}\exp\left(2\gamma\theta/t^{2}\right)\right]$$

$$\mu\left(t\right)=\frac{1}{S\left(t\right)}\left[\log\left(2\right)\right]^{n+1}\left(\gamma\theta\right)^{1/2}\left[\sqrt{\pi}-\frac{t}{\sqrt{2\gamma\theta}}\exp\left(2\gamma\theta/t^{2}\right)\right]-t \tag{14}$$

6. Maximum Likelihood Estimation

In this section, we use maximum likelihood estimation method to estimate the parameters of the NEIR model. Let $X_1, X_2, ...X_n$ be random sample for n observations from NEIR model. The likelihood function of n observations is given by

$$L=\prod_{i=1}^{n}\frac{\gamma\theta\log\left(2\right)2^{1+\exp\left[-\frac{\gamma\theta}{x^{2}}\right]}\exp\left[-\frac{\gamma\theta}{x^{2}}\right]}{x^{3}} \tag{15}$$

The log-likelihood function is given as

$$\log L=n\log\left(\gamma\right)+n\log\left(\theta\right)+n\log\left[\log\left(2\right)\right]+n+\sum_{i=1}^{n}\exp\left(-\frac{\gamma\theta}{x^{2}}\right)\log\left(2\right)$$
$$-\frac{\gamma\theta}{\sum_{i=1}^{n}x^{2}}-\sum_{i=1}^{n}\log\left(x^{3}\right) \tag{16}$$

Differentiating equation (14) with respect to γ and θ, we have

$$\frac{\partial\log L}{\partial\gamma}=-\frac{\theta}{nx^{2}}+n\left(\frac{1}{\gamma}-\frac{\exp\left(-\frac{\gamma\theta}{x^{2}}\right)\theta\log\left(2\right)}{x^{2}}\right) \tag{17}$$

$$\frac{\partial\log L}{\partial\theta}=-\frac{\gamma}{nx^{2}}+n\left(\frac{1}{\theta}-\frac{\exp\left(-\frac{\gamma\theta}{x^{2}}\right)\gamma\log\left(2\right)}{x^{2}}\right) \tag{18}$$

The equations (15) and (16) have no such close form, so we use R software to get numerical solution.

7. Order Statistics

Let $X_1, X_2, ...X_n$ denote the order statistics obtained from a random sample $x_1, x_2, ...x_n$ from a continuous population with cdf $G(x)$ and pdf $g(x)$ as follows

$$g(x_i) = \frac{n!}{(i-1)!(n-i)!} g(x_i)\{G(x_i)\}^{i-1}\{1-G(x_i)\}^{n-i}$$

7.1. PDF of Minimum, Median and Maximum Order Statistics

The pdf of minimum order statistics is given by

$$g(x_1) = n\left[2 - 2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}}\right]^{n-1}\left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]}\exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3}\right] \tag{19}$$

The pdf of median order statistics is given by

$$g(x_{m+1}) = \frac{(2m+1)!}{m!m!}\left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]}\exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3}\right]\left[2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}} - 1\right]^{m} \times \left[2 - 2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}}\right]^{m} \tag{20}$$

The pdf of maximum order statistics is given by

$$g(x_n) = n\left[2^{\exp\left[-\theta\left(\frac{1}{x^2}\right)\right]^{\gamma}} - 1\right]^{n-1}\left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x^2}\right]}\exp\left[-\frac{\gamma\theta}{x^2}\right]}{x^3}\right] \tag{21}$$

7.2. Joint PDF of Minimum and Maximum Order Statistics

The joint pdf of the minimum and maximum, that is, i^{th} and j^{th} order statistics from NEIR model, is given by

$$g(x_i, x_j) = k\left[2^{\exp\left[-\theta\left(\frac{1}{x_i^2}\right)\right]^{\gamma}} - 1\right]^{i-1}\left[2^{\exp\left[-\theta\left(\frac{1}{x_j^2}\right)\right]^{\gamma}} - 2^{\exp\left[-\theta\left(\frac{1}{x_i^2}\right)\right]^{\gamma}}\right]^{j-i-1}\left[2 - 2^{\exp\left[-\theta\left(\frac{1}{x_i^2}\right)\right]^{\gamma}}\right]^{n-j} \times \left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x_i^2}\right]}\exp\left[-\frac{\gamma\theta}{x_i^2}\right]}{x_i^3}\right]\left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x_j^2}\right]}\exp\left[-\frac{\gamma\theta}{x_j^2}\right]}{x_j^3}\right] \tag{22}$$

where

$$k=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}$$

For the joint pdf of minimum and maximum order statistics we put $i=1$and $j=n$ we get

$$g(x_1,x_n)=n(n-1)\left[2^{\exp\left[-\theta\left(\frac{1}{x_n^2}\right)\right]^{\gamma}}-2^{\exp\left[-\theta\left(\frac{1}{x_1^2}\right)\right]^{\gamma}}\right]^{n-2}\times\left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x_1^2}\right]}\exp\left[-\frac{\gamma\theta}{x_1^2}\right]}{x_1^3}\right]\times \left[\frac{\gamma\theta\log(2)2^{1+\exp\left[-\frac{\gamma\theta}{x_n^2}\right]}\exp\left[-\frac{\gamma\theta}{x_n^2}\right]}{x_n^3}\right] \quad (23)$$

8. Simulation

In this section, we conduct simulation study to check the performance of the NEIR estimators. The algorithm is as follows;

- A random sample $x_1,x_2,.....,x_n$ of sizes $n=(30,60,90,150,240,300)$ is generated from the NEIR model.
- Baseline values for the parameters have been selected.
- For each sample size maximum likelihood estimates were computed.
- The simulation procedure is carried out 1000 times for each sample, and the parameters mle's, biases and mean square error (MSE) are recorded.

The mle's biases and MSE for each parameter is calculated as

$$MLE=\frac{1}{1000}\sum_{i=1}^{1000}\hat{\lambda}$$

$$Bias=\frac{1}{1000}\sum_{i=1}^{1000}(\hat{\lambda}-\lambda)$$

and

$$MSE=\frac{1}{1000}\sum_{i=1}^{1000}(\hat{\lambda}-\lambda)^2$$

Table 2 and 3 shows the simulation results and it is clear that;

- MLEs tends to stable
- Biases tends to zero
- MSEs decreases

Table 2: The parameter estimation from the NEIR distribution using MLE

Sample Size	**MLE (θ)**	**Bias (θ)**	**MSE (θ)**	**MLE (γ)**	**Bias (γ)**	**MSE (γ)**
30	0.7994	0.0994	0.7939	1.2151	0.2151	1.1808
60	0.6137	0.1137	0.9252	1.1666	0.0666	0.4683
90	0.6703	0.1703	0.2212	1.1833	0.2833	0.3061
150	0.6129	0.2129	0.8444	1.0778	0.0778	0.6399
240	0.6949	0.1949	0.7996	1.0886	0.2886	0.5290
300	0.5836	-0.0164	0.0338	1.0552	0.3552	0.0342

Table 3: The parameter estimation from the NEIR distribution using MLE

Sample Size	**MLE (θ)**	**Bias (θ)**	**MSE (θ)**	**MLE (γ)**	**Bias (γ)**	**MSE (γ)**
30	1.2498	0.4498	4.0223	1.3748	0.0748	0.9333
60	1.1287	0.3287	2.3245	1.3493	0.1493	0.3769
90	0.9648	0.1648	1.1454	1.3977	0.1977	0.8158
150	0.9445	0.3745	0.3838	1.2567	0.1007	0.9559
240	0.8806	0.0806	0.8858	1.2278	0.0978	0.9217
300	0.8103	-0.0097	0.0432	1.2170	0.2970	0.0046

9. Application:

In this section, we use a real data to compare the performance of the NEIR model with other competitive models, namely: Rayleigh Distribution (Rey), Exponential Distribution (Exp), Generalize exponential distribution. (GExp) and Arcsine Exponential distribution (AsExp). To compare the fit of all models we consider Akaike information criteria (AIC), Bayesian information criteria (BIC), corrected information criteria (CAIC), Hannan-Quinn information criteria (HQIC) and Kolmogorov Smirnov (K-S) statistic and its p-value.

Data: Badar and Priest (1982) discussed the real dataset consists of 69 samples observed failure times, the dataset represents GPA measurements taken from individual carbon fibers and from impregnated tows consisting of 1000 fibers. **Data:** 0.562, 0.564, 0.729, 1.216, 1.474, 1.632, 1.816, 2.020, 2.317, 1.247, 1.490, 1.879, 2.059, 2.346, 0.950, 1.277, 1.522, 1.728, 1.883, 2.068, 2.378, 1.053, 1.305, 1.524, 1.740, 1.892, 2.071, 2.483, 1.111, 1.348, 1.552.204,2.835,1.208,1.429,1.632, 1, 1.764, 1.934, 2.130, 2.835, 1.676, 1.824, 2.023, 2.334, 1.256, 1.503, 1.684, 1.836, 2.050, 2.340, 0.802, 1.271, 1.520, 1.685 ,1.115, 1.313, 1.551, 1.761, 1.390,1.609,1.785,1.947 ,1.804,1.976, 2.262, 1.898, 2.098, 2.683, 1.194.

Further the graphical display of the data provided in **Figure 5.** A visual comparison is also provided using fitted pdf and fitted cdf given in **figure 6.** The goodness of fit for the data are given in **Table 5.** The maximum likelihood estimates of the fitted distributions are given in **table 4.**

(a) (b)

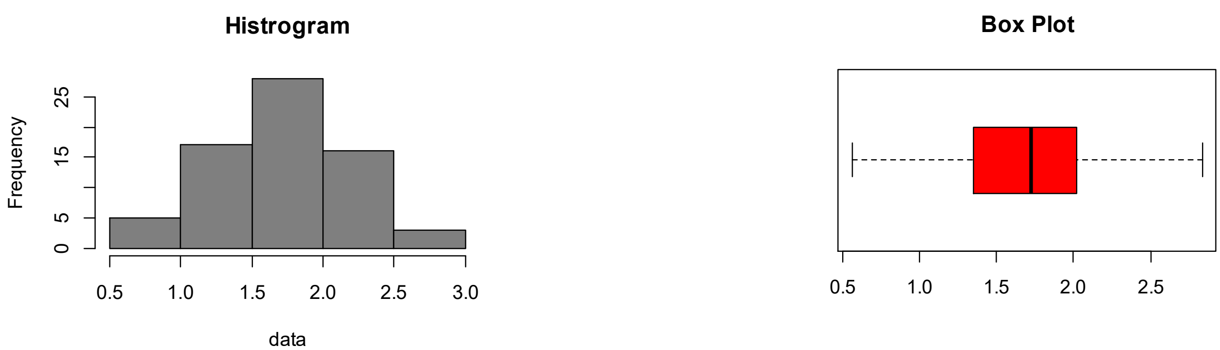

Figure 5: (a) Histogram (b) Boxplot. A graphical display of the data

(a) (b)

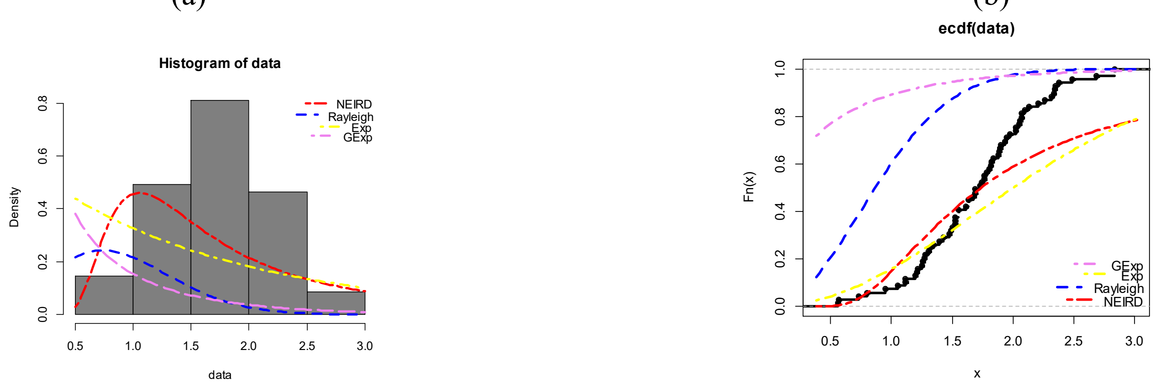

Figure 6: (a) Fitted pdfs (b) fitted cdfs for the data

Table 4: Maximum likelihood estimates of the fitted distributions

Models	$\hat{\theta}$	$\hat{\gamma}$
NEIR	1.465762	1.105227
Ray	1.35752	-
Exp	0.58775	-
GExp	1.985368	0.83664
AsExp	1.079901	-

Table 5: AIC, BIC, CAIC, HQIC, Ks and p-values for the data

Models	AIC	BIC	CAIC	HQIC	Ks	p-value
NEIR	190.628	195.0962	190.8098	192.4007	0.25918	0.000188
Ray	418.5834	420.8175	418.6431	419.4698	0.61521	2.2e-16
Exp	213.3396	215.5737	213.3993	214.2259	0.39256,	1.2e-09
GExp	215.3396	219.8078	215.5214	217.1123	0.3926	0.6067
AsExp	260.4484	262.6825	260.5081	261.3347	0.50655	8.372e-16

10. Conclusion

In this study, we introduced and examined the New Exponentiated Inverse Rayleigh (NEIR) distribution, an extension of the inverse Rayleigh distribution utilizing the exponentiation technique. The NEIR model offers enhanced flexibility for modeling lifetime data, particularly those exhibiting right-skewed behavior. We derived and analyzed its key statistical characteristics, including the probability density function, cumulative distribution function, survival and hazard functions, moments, entropy measures, and order statistics. The parameter estimation was addressed using the maximum likelihood method, supported by a simulation study that confirmed the estimators' consistency and efficiency. Furthermore, the application of the NEIR distribution to real-world data demonstrated its superior fit compared to several competing models, as indicated by lower values of AIC, BIC, CAIC, HQIC, and the Kolmogorov–Smirnov statistic. Overall, the NEIR distribution provides a valuable addition to the family of lifetime distributions, with strong theoretical foundations and practical effectiveness. Future work could explore Bayesian estimation techniques, multivariate extensions, or potential applications in reliability engineering, biostatistics, and risk assessment.

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