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Fourier Autoregressive Moving Average Model for Complex Time Series Datasets

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ABSTRACT

Time series datasets that are non-stationary and seasonal are often analysed using various techniques. Due to structural changes, many time series datasets have become increasingly complex. Therefore, there is a need to develop a model capable of analysing irregular variation, cyclical, seasonal and periodic variation in time series datasets. The Fourier Autoregressive Moving Average model (FARMA) was developed to analyse complex time series datasets. Time plots were used to determine the variation pattern, correlogram and magnitude plots were used to identify model order and estimation was carried out using Maximum likelihood (ML) estimation method, Diagnostic checking was achieved base on the Durbin Watson statistical test, histogram, Autocorrelation function (ACF) and Partial Autocorrelation (PACF) plot of the residuals. The model was validated using coefficient of determination (R^2) and adjusted coefficient of determination ($\overline{R}^2$) while forecast evaluation was based on the least values of Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE). The Nigerian exchange rate was considered and the time plot indicated the presence of trend, cyclical, seasonality and irregular variations simultaneously. The FARMA model was estimated using ML estimation method for fitting the coefficient of the model. The model were diagnosed using Durbin Watson statistic, autocorrelation and partial autocorrelation function, histogram of the residual. The results obtained indicated that the Fourier Autoregressive moving average model performed better than ARMA(2,2,3), SARIMA(2,2,1)(2,2,3)$_{12}$ and FAR(6) models in terms of coefficient of Determination and Adjusted coefficient of Determination (0.9650,0.9574,0.7825,0.7975) and (0.9610,0.9540,0.7729 0.7934) for Nigeria exchange rate The optimum FARMA component model was established using Akaike information Criteria(AIC) and Bayesian information criteria (BIC) values with (10.3948 and 10.674) for Nigeria exchange rate which signifies the minimum value of the information criteria. The efficiency of the FARMA(6) model was ascertained by modeling and forecasting Nigeria exchange rate datasets. The forecast evaluation of the proposed model was (155.0275, 263.3046 and 42.1168) for MAE, RMSE and MAPE respectively for Nigeria exchange rate, the smallest value of FARMA(6) model makes it better model compared with existing fitted model.
Therefore, the proposed FARMA model is capable of modelling and forecasting time series datasets that exhibits seasonal, cyclical and periodic and irregular structure simultaneously. Fourier autoregressive moving average model is applicable in other field like environmental data, climatic data and sales data. It is recommended that the model can be applied in other sphere of life.

Keywords: *Variations, Model, Fourier, cyclical, periodic, model evaluation, diagnostic checking*

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1. Introduction

In analyzing time series datasets, it is imperative to determine the kind of variation the series exhibited which has to be considered to determine the efficiency and the forecast that will be obtained [6]. Complex variation has been identified in a time series datasets when the data does not follow simple or constant patterns, it reflects a mix of components like trends, cyclic, seasonality and random error simultaneously. Several components of time series analysis have been identified, including trend, seasonal, cyclical, irregular variation or random error. [19].The time series components has been analyzed using several techniques [20]. Traditional method (Box and Jenkins) approach have yielded positive result in modelling and forecasting time series datasets [1, 2]. Additionally, a number of techniques have been developed to enhance time series dataset analysis, particularly in univariate and multivariate models. Multi-unit time series designs, multivariate time series designs, and pattern analysis are recent extensions of time series techniques. [3]. Their application have been applied in various fields like environmental, climatic, trade and commerce also in agriculture,[8] but due to structural changes which can be attributed to several factors like economic recession, government policy, pandemic, social and environmental factors which makes time series datasets to be complex [9]. Most time series datasets now exhibit more than two variations, several time series model like autoregressive (AR) autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA) among others have been used to analyse time series datasets with different variation over the years, a critical analysis showed that mostly used models do not have capabilities of handling complex variation time series datasets [10].

This research article aims to develop a time series approach for complex structured in Nigeria economic datasets since the degradation in economy has been attributed to unstable exchange rate [13] Several statistical approach like Autoregressive, Moving Average, ARMA, Seasonal Autoregressive Moving Average (SARIMA) had been used to model the presence of one or two variations in a time series datasets not considering the datasets that exhibit more than two variations especially the presence of irregular variations. Hence, a Fourier autoregressive moving Average model (FARMA) that combine Fourier terms with autoregressive moving average model developed for taking care of cyclical, seasonality and irregular variations as well as interdependency between the time series and its lags, also the challenges of overfitting commonly neglected in time series modelling with Fourier terms will be handle by identifying the model to be estimated using spectral density function which will be used to model and forecast Nigeria Exchange rate datasets from January 2004 to October 2024.

Based on scientific literature, temperature fluctuations particularly rising temperatures due to climate change, pose significant threats both locally in Nigeria and worldwide [16]. Globally, increased temperatures contribute to more frequent and severe weather events such as droughts, floods and storms disrupting ecosystem, economies and communities [17]There has been more concentration on enhancing forecasting methods to identify various time series patterns. It was discovered that time series procedures like ARIMA could solve complex time series pattern [5] but was deduced that it would be effective for short term forecast. [18] employed SARIMA to analyse rainfall pattern and diagnostic test confirmed the adequacy of the model.[4] discussed the evolution of ARIMA through three different stages and highlighted its hybrid modeling applications with a special focus on environment, health, and air quality. They concluded that hybrid models are more robust and better able to uniformly capture all patterns of the series, demonstrating their higher potential in this field.

To analyse rainfall patterns and identify the periods of highest and lowest rainfall at the stations under investigation [7] applied Fourier series to daily rainfall data and the driest and wettest period were projected. Hence, it was discovered that the irregular variation presence in time series datasets were not captured by traditional method

2. Methodology

Fourier Autoregressive Moving Average Model (FARMA)

Fourier Autoregressive Moving Average (FARMA) model is given as

$$X_{kw+v} = \delta + \sum_{i=1}^{p} [acos\omega + bsin\omega] X_{kw+v-} + c \sum_{j=1}^{q} [a_j cos\omega + b_j sin\omega] \varepsilon_{kw+v-} + \mu_{kw+v} \quad (1)$$

where v is the period index $(v = 1, 2, \ldots, n)$, k is the year index $(k = 0 \pm 1, \pm 2, \ldots)$, a, b, a_j and b_j are the Fourier autoregressive and moving average coefficients, w is the number of season, μ_{kw+v} is a white noise with mean zero (0) variance ${\sigma_\varepsilon}^2$

Model building in Fourier Auoregressive Moving Average Model

The four basic steps in analysing Fourier Autoregressive moving Average model building will be carried out as follows.

Model Identificaion for Fourier Autoregressive moving Average model

The model will be identified using time plot, Autocorrelation function (ACF), Partial autocorrelation function as well as spectral density function which are given as

For a univariate series X_{kw+v} in which the white noise term μ_{kw+v} were assumed to be independent, the autocorrelation function is defined as

$$\gamma_{kw+v} = cov(X_{kw+v} X_{kw+v-i}) = E[(X_{kw+v} - \mu_{V)}(X_{kw+v-i} \text{-} \mu_{v-i})] \quad (2)$$

For the season v at backward lag l ≥ 0, then the ACF for the period v at backward lag lag l ≥ 0 is defined as

$$\rho_{i(v)} = E[(\frac{X_t - \mu_{V)}}{\sqrt{\gamma_{0(v)}}} \left(\frac{X_{t-i} - \mu_{v-i}}{\sqrt{\gamma_{(v-i)}}}\right)][E[(X_t, X_{t-i}] \quad (3)$$

$$= \frac{\gamma_{t(v)}}{\sqrt{\gamma_0(v)\gamma_0(v-l)}} \quad (4)$$

The partial autocorrelation function $\varphi_{ii}(v)$ can be applied to determine the exact relationship between $X_t \, and X_{t-i}$ after removing the effect of the intervening observation and it's defined for integers $l \geq 1$ as

$$\varphi_{ii} = corr\left[\frac{X_t X_{t-i}}{X_{t-i} X_{t-i}}\right] \quad (5)$$

Where partial correlation coefficient is denoted as φ_{ii}

Therefore

$$\varphi_{ii} = \frac{cov[X_t - \hat{X}_t)([X_{t-i} - \hat{X}_{t-i})}{\sqrt{var(X_t - \hat{X}_t)([X_{t-i} - \hat{X}_{t-i})}} \quad (6)$$

And

Spectral density function $f(\omega)$ is given as

$$f(\omega) = \frac{1}{2\pi} \sum_{t=0}^{\pi} \gamma_t : \omega \, \epsilon \, [-\pi, \pi] \quad (7)$$

Model Estimation of Fourier Autoregressive Moving Average Model

The Fourier Autorgressive moving Average Model will be eaimated using maximum likelihhod estimation method and can be expressed as

$$L = (\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}} (X_{kw+v} - \delta - \sum_{i=j=1}^{p,q} [acos\omega X_{kw+v-} - bsin\omega X_{kw+v-i} - a_j cos\omega \varepsilon_{kw+v-j} - b_j sin\omega \varepsilon_{kw+v-}]^2 \quad (8)$$

After the estimation of the model, the better model will be chosen based on the minimum values of Akaike information criteria (AIC) and Bayesian information criteria (BIC)

$$AIC(M) = n.ln\hat{\sigma}^2{}_{\varepsilon}(v) + 2M(v) \quad (9)$$

$$BIC = n.ln\hat{\sigma}^2{}_{\varepsilon}(v) + k.\ln(n) \quad (10)$$

Where $\hat{\sigma}^2{}_{\varepsilon}(v)$ is the error variance and n is the number of number Fourier Autoregressive Moving Average coefficients in the season respectively.

Model Validation

The model will be validated based on the value of Coefficient and Adjusted Coefficient of Determination

Coefficient of Determination for FARMA Model

The coefficient of Determination for FARMA is obtain using

$$R^2 = \frac{(\hat{\boldsymbol{B}})'(X'X)\beta - n\bar{y}^2}{y'y - n\bar{y}^2} \quad (11)$$

Adjusted Coefficient of Determination for FARMA

The adjusted coefficient of Determination can be obtained as

$$\bar{R}^2 = 1 - \frac{n-1}{n-k}(1 - R^2) \quad (12)$$

Diagnostic Checking in Fourier Autoregressive Moving Average Model

After a time series model has been specified and its parameter have been estimated, there is need to check to test whether the or not the original specification is correct. Hence, a careful analysis of the estimated residuals was carried out by checking whether the residual are white noise and it's done by computing the sample ACF and PACF of the residual the sample autocorrelation of the residual $\hat{\Gamma}_i(\varepsilon)$ would be distributed approximately normally about zero and variance and statistically significant within two standard deviation with α =0.05. If the model has been correctly specified, the residual $\hat{\Gamma}_i(\varepsilon)$ should resemble the white noise process.

Forecasting Evaluation

Once forecast is obtained, an evaluation is computed to determine if the actual value of the series forecast are observed. In achieving this, there are some measurements of the accuracy of forecast that will be applied, these are Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE)

$$RMSE = \sqrt{\frac{1}{t+1}\sum_{t=1}^{p-1}(\widehat{X}_t - X_t)^2} \quad (13)$$

$$MAPE = \left| \sum_{t=1}^{p-1} \frac{\hat{X}_t - X_t}{\hat{X}_t} \right| \quad (14)$$

$$MAE = \frac{1}{t+1} \sum_{t=1}^{p-1} \left| \hat{X}_t - X_t \right| \quad (15)$$

$where\ t = 1,2, \ldots..p-1$. The exact value and predicted values for corresponding t values are denoted by $\hat{X}_t and\ X_t$ respectively. The smaller the values of RMSE, MASE, MAE, the better the forecasting performance of the model.

Error term Performance based on Durbin Watson Statistic

If the ε_t is the residual for the observation at time t, then the test statistic is given as

$$d = \frac{\sum_{t=2}^{T} (\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{T} {\varepsilon_t}^2} \quad (16)$$

Where T is the number of observations. If the sample size is large, then this can be linearly mapped to the time series data The Durbin Watson statistic ranges from 0 to 4. As d = 2(1 – r) where r is the residual sample autocorrelation, a value close to 2 indicates no autocorrelation, while values less than 2 suggest positive autocorrelation and the value greater than 2 suggest negative autocorrelation.

3. Results and Discussion

In order to determine the efficiency of the FARMA model, a visual observation of the Nigeria monthly exchange rate time plot in Figure 1 from January 2004 to October 2024 was analysed which exhibits trends and cyclical variations as a result of rise and fall observed in the time series plot. The optimal model were chosen based on the minimum value of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) given in the result below.

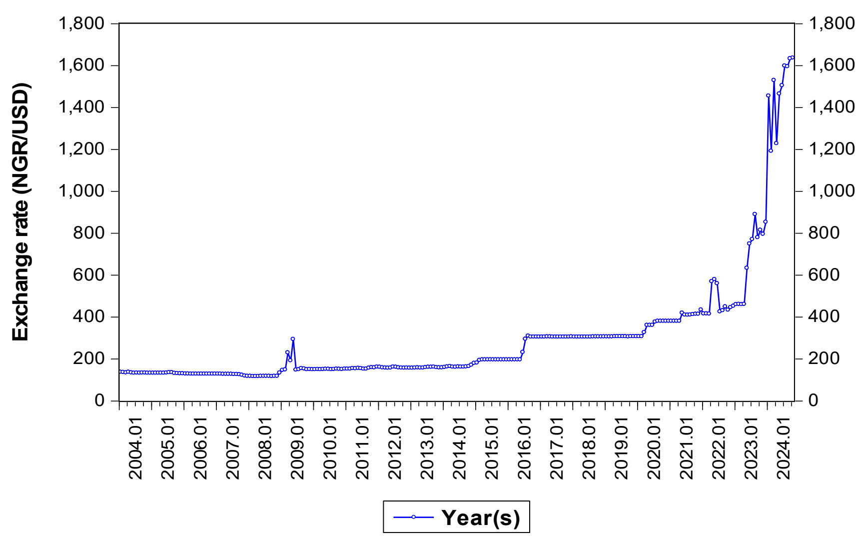

Figure 1: Time plot of Nigeria Exchange Rate from 2004 to 2024

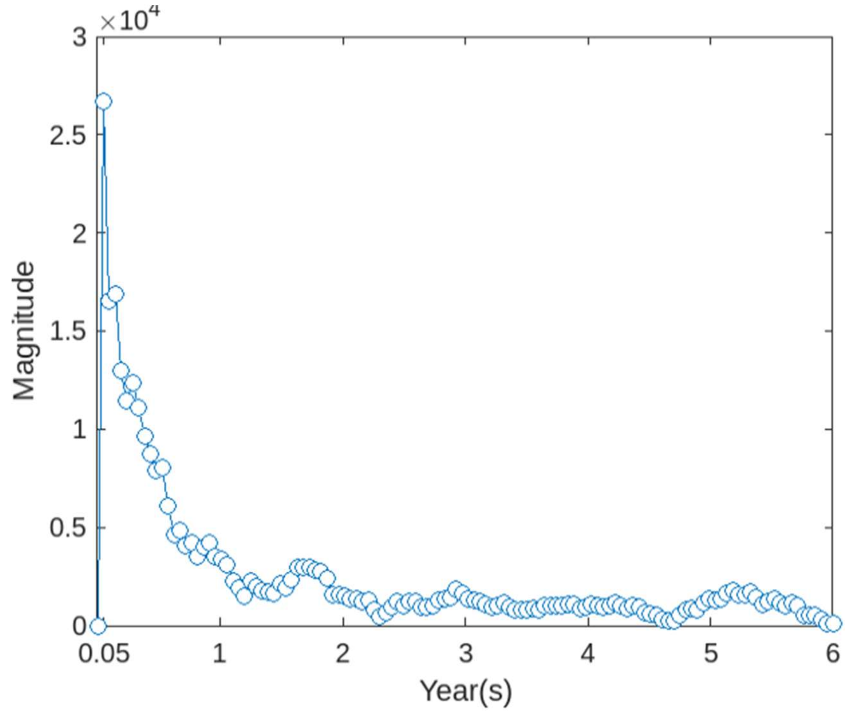

Figure 2: Magnitude plot of Nigeria Exchange Rate from 2004 to 2024

In order to avoid over fitting that that often occur using Fourier, the magnitude plot Figure 2 depicts six components to be estimated

Table 1: Autocorrelation Plot of the Residual for Fourier Autoregressive Moving Average Model (Nigeria Exchange Rate)

Table 1 reveals the ACF and PACF plot of the series which shows that the series exhibit seasonality a result of the pattern shown.

The FARMA(6) model was estimated using maximum likelihood model as shown in Equation 1

$$X_t = 861.27 + 0.33 \cos\frac{2\pi}{12}X_{t-1} + 1.7919 sin\frac{2\pi}{12}X_{t-1} + 0.4199 cos\frac{2\pi}{12}\epsilon_{t-1} - 1.1109 sin\frac{2\pi}{12}\epsilon_{t-1}$$

$$-2.1576 \cos\frac{4\pi}{12}X_{t-2} - 0.6077 sin\frac{4\pi}{12}X_{t-2} 0.4818 cos\frac{4\pi}{12}\epsilon_{t-2} + 0.6042 sin\frac{4\pi}{12}\epsilon_{t-2} -$$

$$0.3808 \cos\frac{6\pi}{12}X_{t-3} - 0.5029 sin\frac{6\pi}{12}X_{t-3} + 0.1339 cos\frac{6\pi}{12}\epsilon_{t-3} - 0.6218 sin\frac{6\pi}{12}\epsilon_{t-3} -$$

$$9.2394 \cos\frac{8\pi}{12}X_{t-4} - 0.2171 sin\frac{8\pi}{12}X_{t-4} + 1.7758 cos\frac{8\pi}{12}\epsilon_{t-4} + 0.9984 sin\frac{8\pi}{12}\epsilon_{t-4} +$$

$$0.4456 \cos\frac{10\pi}{12}X_{t-5} + 1.3711 sin\frac{10}{12}X_{t-5} + 2.9156 cos\frac{10}{12}\epsilon_{t-5} - 0.5754 sin\frac{10}{12}\epsilon_{t-5} -$$

$$1.4962\cos\frac{12\pi}{12}X_{t-6} - 0.8362sin\frac{12}{12}X_{t-6} + 0.3399cos\frac{12}{12}\epsilon_{t-6} - 0.0356sin\frac{12\pi}{12}\epsilon_{t-6} \quad (1)$$

with R^2= 0.9651, Adjusted R^2= 0.9611, Durbin Watson =1.9989, AIC = 10.3948, BIC = 10.7676

Table 2: ACF and PACF of Residual For Six Component FARMA Model

Autocorrelation	Partial Correlation		AC	PAC	Q-Sta...	Prob...
		1	-0.00...	-0.00...	0.0003	
		2	-0.02...	-0.02...	0.1264	
		3	0.006	0.006	0.1348	
		4	0.002	0.001	0.1357	
		5	0.006	0.006	0.1435	
		6	-0.04...	-0.04...	0.5871	
		7	0.037	0.037	0.9373	
		8	0.054	0.052	1.6701	
		9	0.048	0.050	2.2473	
		1...	0.015	0.017	2.3013	
		1...	0.031	0.034	2.5524	
		1...	0.062	0.061	3.5529	
		1...	0.041	0.046	3.9873	0.046
		1...	-0.03...	-0.02...	4.2765	0.118
		1...	-0.01...	-0.01...	4.3563	0.225
		1...	0.039	0.032	4.7478	0.314

Table2. reveals the residual of the fitted FARMA(6) model were diagnosed with residual ACF and PACF which reveals the model falls within confidence bound of $\alpha = 0.05$,therefore the residual white noise

Table 3: Value of Coefficient of Determination and Adjusted Coefficienof Determination for Nigeria Exchange Rate

Model	Coefficientof Determination R^2	$Adjusted$ Coefficient of Determination R^2
ARMA	0.7975	0.7934
SARIMA	0.7825	0.7729
FAR	0.9574	0.9540
FARMA	0.9650	0.9610

Based on the result in Table 3. it was reveal that FARMA(6) model explain the variation up to 96% in the Nigeria Exchange Rate and also have a good predictive value

Table 4. Value of Akaike Information Criterion (AIC) Bayesian Information Crierion (BIC) of Nigeria Exchange Rate

Model	AIC	BIC
ARMA	10.53	10.68
SARIMA	10.81	10.97
FAR	10.39	10.78
FARMA	10.17	10.67

FARMA(6) also exhibit a minimum value based on AIC and BIC value which makes better than the existing model

Table 5 Forecast Evaluation for Nigeria Exchange Rate of FARMA Model

Model	MAE	RMSE	MAPE
ARMA	211.26	372.93	50.87
SARIMA	876.26	1120.17	280.19
FAR	496.26	557.33	288.87
FARMA	155.07	263.30	42.12

Based on the comparison ARMA(2,2,3), SARIMA(2,2,1)$(2,2,3)_{12}$ and FAR(6) could not capture the presence of irregular flunctuation in the Nigeria Exchange Rate. Therefore, Fourier Autoregressive Moving Average Model FARMA is more suitable for modelling and forecasting Nigeria Exchange Rate.

4. Conclusion

The Fourier Autoregressive Moving Average Model was used to analyse Nigeria exchange rate from 2004 to 2024. FARMA(6) was identified based on ACF and PACF plot and spectral density function. The coefficients of the model were estimated using Maximum likelihood estimation. The optimal model FARMA(6) model was chosen based on minimum value of AIC and BIC. The residual was diagnosed to be white noise. The model was compared with some existing model ARMA(2,2,3), SARIMA(2,2,1)$(2,2,3)_{12}$ and FAR(6) based on model validation and forecast evaluation and none could capture the presence of irregular variation nor the presence of seasonality and perodicitty in Nigeria exchange rate datasets simultaneously. SARIMA(2,2,1)$(2,2,3)_{12}$ only capture the seasonality while FAR(6) capture the presence of seasonality and periodicity in Nigeria exchange rate, FARMA(6) was able to determine the presence of seasonality, periodicity as well as random error in Nigeria exchange rate datasets. Therefore, FARMA(6) model was suitable as it reveals that unit increase in time will cause rise or fall in Nigeria exchange rate and the adverse have impact on Nigeria Economy. So Government should make policy that will strengthening Naira against foreign currencies in order to enjoy stable economy.

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