Journal of Probability and Statistical Science
24(1), 1-15 Feb. 2026

Visualizing the Mean, Mean Deviation, and Standard Deviation from the Disc Plot of a Discrete Random Variable

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ABSTRACT

Sarkar and Rashid (2024) visualized the joint probabilities of two discrete random variables using a 2D disc plot from which they obtained graphically the conditional distributions of Y given $X = a$, and X given $Y = b$; and the marginal distributions of X, Y, $X + Y$, $X - Y$ and $cX + dY$. Here, we visualize the mean, mean deviation, and standard deviation of each discrete random variable starting from its disc plot. Geometric visualizations help users better comprehend these concepts.

Keywords: joint distribution, disc plot, weighted average, probability mass function

1. Introduction

Joint distributions of two or more random variables (RVs) frequently arise in probability and statistics. For a single discrete RV (DRV), its probability mass function (PMF) is typically depicted using a 2D stick plot showing the value along the x-axis and the associated probability $p_X(x)$ as the height of a stick standing at x parallel to the y-axis. See Sarkar and Rashid (2019). Furthermore, they depict the mean and the standard deviation (SD) using a single-headed arrow, starting at the mean and having a length equal to the SD, under the PMF.

Recall that the mean of any DRV is $\bar{x} = \sum x \cdot p\,(x)$, the mean deviation (MD) is $\bar{d} = \sum |x - \bar{x}| \cdot p\,(x)$, the mean square is $E[X^2] = \sum x^2 \cdot p(x)$, the variance is $\sigma^2 = \sum (x - \bar{x})^2 \cdot p\,(x) = E[X^2] - \bar{x}^2$, and the SD is $\sigma = \sqrt{\sigma^2}$. To visualize the mean, the MD, and the SD for a DRV represented by its cumulative distribution function (CDF), see Sarkar and Rashid (2019). To visualize the same quantities for a data set, see Sarkar and Rashid (2015, 2016).

To visualize two DRVs simultaneously, one may consider a bivariate stick diagram — a 3D picture showing the values of X and Y (on the xy-plane) and the associated probability $p(x, y)$ as the height of the stick orthogonal to the xy-plane (or parallel to the z-axis). See Wackerly, *et al.* (2014). However, recognizing the difficulty many students face in understanding and further processing 3D pictures, Sarkar and Rashid (2024) proposed displaying the joint PMF of two DRVs using a disc plot, in which each disc is centered at the vector (x, y) and has an area proportional to the joint probability $p(x, y)$ at (x, y). Further, they obtain the conditional distributions of Y given $X = a$, and X given $Y = b$, and the marginal distributions of X, Y, $X + Y$, $X - Y$ and $cX + dY$. In particular, for each RV, the mean and standard deviation (SD) were computed externally and then depicted as an arrow with its tail representing the mean and length representing the SD. Here, starting from the disc plot of each DRV, we visualize the mean, the MD, and the SD geometrically, even when the values of $x,\ y,\ p(x, y)$ are not disclosed. To do so, we first need the concept of the weighted average of two points.

All pictures are drawn using the freeware R. See R Core Team (2024).

- Received July 29, 2025, in final form February 2026.
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2. Weighted Average

The weighted average of two points with weights proportional to their associated probabilities p and q (represented by discs centered at the points and of radii proportional to $\sqrt{p}$ and $\sqrt{q}$, so that the disc areas are proportional to p and q) is shown in Figure 1 and described below.

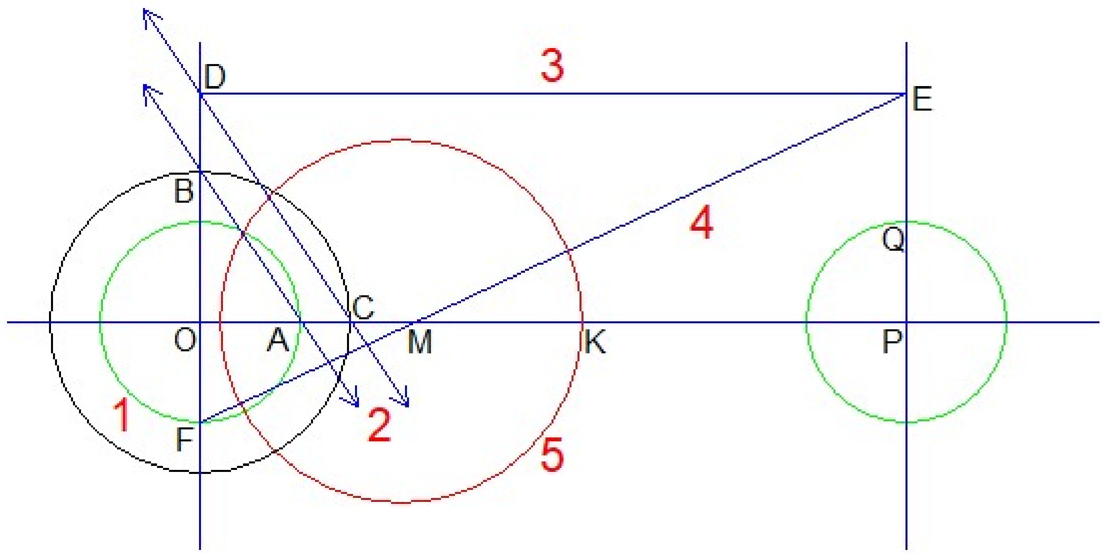


Figure 1: The weighted average of two points O and P, with weights p and q, proportional to the areas of the circles of radii OB=$\sqrt{p}$ and PQ=$\sqrt{q}$, is a point M with associated weight $p+q$

Method of Finding the Weighted Average

The following steps construct the weighted average of two points, O and P, with weights p and q, proportional to the areas of the circles of radii OB=$\sqrt{p}$ and PQ=$\sqrt{q}$, respectively.

1. Join OP and draw perpendiculars OB=$\sqrt{p}$ and PQ=$\sqrt{q}$ on the same side of OP. Draw another disc with center O and radius OA=OF=PQ=$\sqrt{q}$, where A is on OP and F is on extended BO. Let C be on OP such that OC=OB=$\sqrt{p}$.
2. Join AB. Draw CD parallel to AB with D on extended OB. Then OD:OC=OB:OA, implying that OD$= p/\sqrt{q}$
3. Draw DE parallel to OP with E on extended PQ. Then PE=OD$= p/\sqrt{q}$.
4. Join EF, cutting OP at M. Then OM:MP=OF:PE=q:p, implying that $p \cdot$ OM=$q \cdot$ PM.
5. Draw a disc with center M and radius MK=AB, where K is on OP. By the Pythagorean theorem, we have
$MK^2 = AB^2 = OB^2 + OA^2 = OB^2 + PQ^2 = p + q$.

Thus, M is the weighted average of O and P with weights p and q proportional to the areas of discs (O, OB) and (P, PQ), respectively. The weight of the disc (M, MK) is $p+q$.

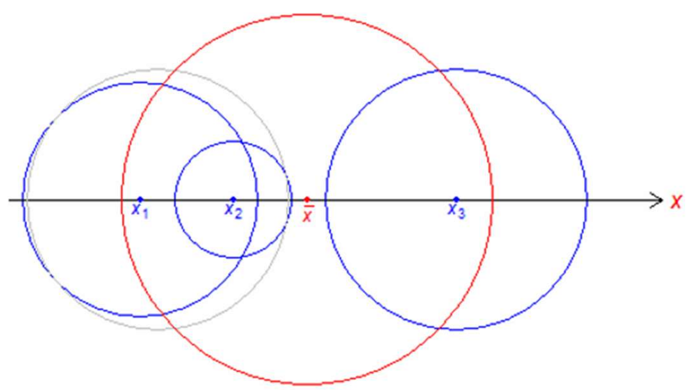
Next, we can combine the weighted average of two points with a third point (along with its associated weight) to obtain the weighted average of all three points, where the weight is given by the sum of the individual weights. Alternatively, we can first compute the weighted average of the first and third points and then take its weighted average with the second point. Equivalently, we can first compute the weighted average of the second and third points and then take its weighted average with the first point.

Likewise, given four points, we can get the weighted average of points one and two, then take the weighted average of points three and four, and finally take the weighted average of these intermediate weighted averages. That is, we can take the weighted average of pairs of points in any order until we obtain the weighted average of any finite number of points with a total weight of one (the sum of probabilities of all points).

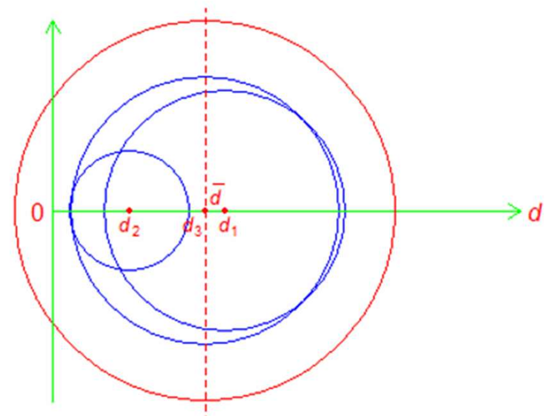
Remark 1: The points do not have to be collinear for the weighted average method to work. They can be points in two or higher dimensions.

3. Mean, MD, and SD of a DRV

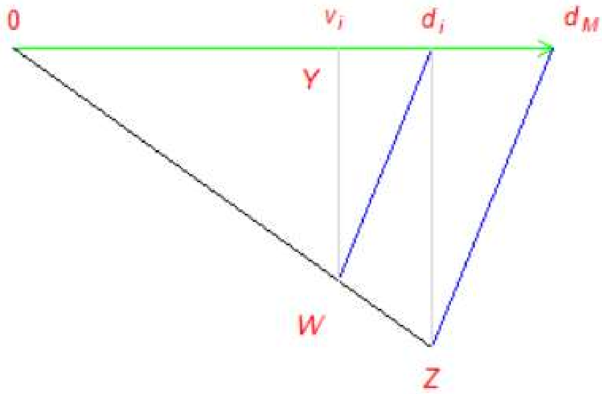
Suppose a DRV X takes m values (centers of discs) $\{x_1, x_2, \dots, x_m\}$ with associated probabilities (proportional to the areas of the discs) $(p_1, p_2, \dots, p_m)$ such that $p_1 + p_2 + \dots + p_{\mathrm{m}} = 1$. Figures 2 (a)-(e) explain the steps to visualize the mean, the MD, and the SD.



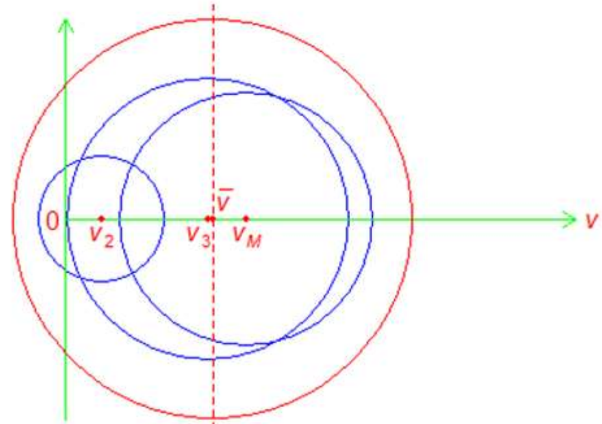
2(a)



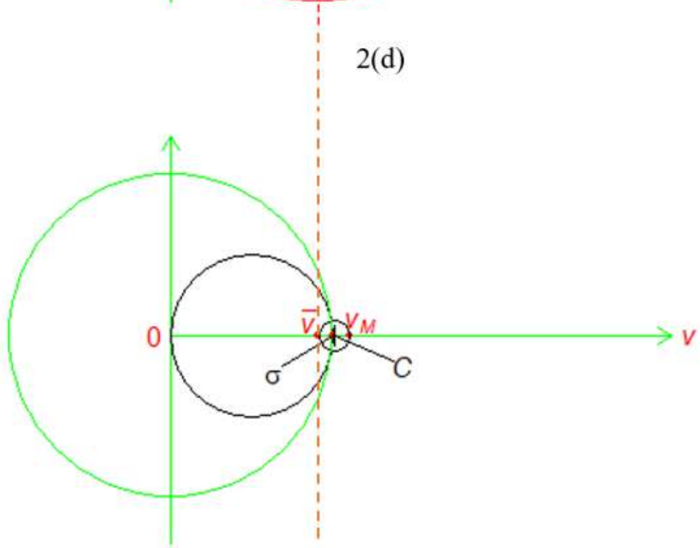
2(b)



2(c)



2(d)



2(e)

Figure 2: Given $m = 3$ points with associated probabilities, find the mean, the MD, and the SD.

a) The mean $\bar{x} = \sum x \cdot p$ is the weighted average of the values (centers of the discs) with associated probabilities (proportional to the areas of the discs) as weights. See Figure 2(a).
b) Having found the mean $\bar{x}$, we draw a vertical line l ($x = \bar{x}$) through the mean and reflect about l the discs on the left of l so that the reflections fall on the right of l. Now the discs (or reflections) are $d_i = |x - x_i|$ units to the right of l ($d = 0$). The weighted average of all discs (or reflections) to the right of l is at a distance $\bar{d} = \sum d \cdot p$ (the MD) to the right of l. See Figure 2(b).
c) Keeping the center of the rightmost disc (say, at d_M) intact, move the center of any other disc from d_i to $v_i = d_i^2/d_M \le d_i$. How to do this is explained in Figure 2(c): Take any point Z not on the d-axis (we recommend taking Z vertically below d_i), and join it to 0, d_i and $v_M = d_M$. Then through d_i draw a line parallel to Zv_M, cutting Z0 at W. Next, through W, draw a line parallel to Zd_i, cutting $0d_i$ at $Y = v_i = d_i^2/v_M$.
d) Apply the technique of Figure 2(c) to all the centers d_i of the discs so they move left to $0 \le v_i = d_i^2/d_M \le v_M = d_M$. Take the weighted average of all the relocated discs (with their centers to the right of l). Call it $\bar{v} = \sum (d_i^2/v_M) \cdot p$. See Figure 2(d). Then, $\bar{v} < \bar{d}$ since $v_i \le d_i$ for every i.
e) Draw the black circle with a diameter on the horizontal axis from $\bar{v}$ to v_M. Let the arithmetic mean of $\bar{v}$ and v_M be C=($\bar{v}$ +v_M)/2. Draw a circle with a diameter of 0C, intersecting the green circle at two points. See Figure 2(e). The distance from the origin to either point of intersection is the geometric mean of $\bar{v}$ and v_M, and is the SD. That is,
$$\sigma = \sqrt{\bar{v} \cdot v_M} = \sqrt{\sum d_i^2 \cdot p}.$$
Admittedly, in Figure 2(e), it is hard to distinguish between the arithmetic mean C and the geometric mean σ of $\bar{v}$ and v_M. Readers may refer to Figure 3 for a clearer demonstration of these inequalities. Whereas $\bar{v} < \bar{d}$, we have $\bar{d} < \sigma < C$.

EXAMPLE 1

A teacher brought ten paperback books to distribute as prizes to her students. She had a copy of *The Story of Pie*, two copies of *The Man Who Knew Infinity*, three copies of *The Joy of Mathematics*, and four copies of *Problem Solving*. The books weigh 320, 530, 290, and 380 grams, respectively. Table 1 shows the PMF of the weight (after sorting). To find the mean, MD, and SD of the weights of the ten books, we would typically proceed with the computations shown in Table 1. Alternatively, having mastered the techniques of this paper, one can geometrically visualize the quantities mentioned above, as shown in Figure 3. Note that the SD is no smaller than the MD.

Table 1: Using the PMF of the weight of a book (X), we compute the mean, MD, and SD

$Books$	# copies	p_i	x_i	d_i	d_i^2/d_M
The Joy of Mathematics	3	.3	290	87	49.47
The Story of Pie	1	.1	320	57	21.24
Problem Solving	4	.4	380	3	0.06
The Man Who Knew Infinity	2	.2	530	d_M =153	153.00
Dot products		$\mathbf{p.1}$	$\mathbf{p.x}$	$\mathbf{p.d}$	$\mathbf{p.d^2/d_M}$
Values of dot products		**1**	**377**	**63.6**	**47.60**
The mean, MD, and SD			$\bar{x}$	$\bar{d}$	$\sigma = \sqrt{47.60 * 153}$ $= 85.34$

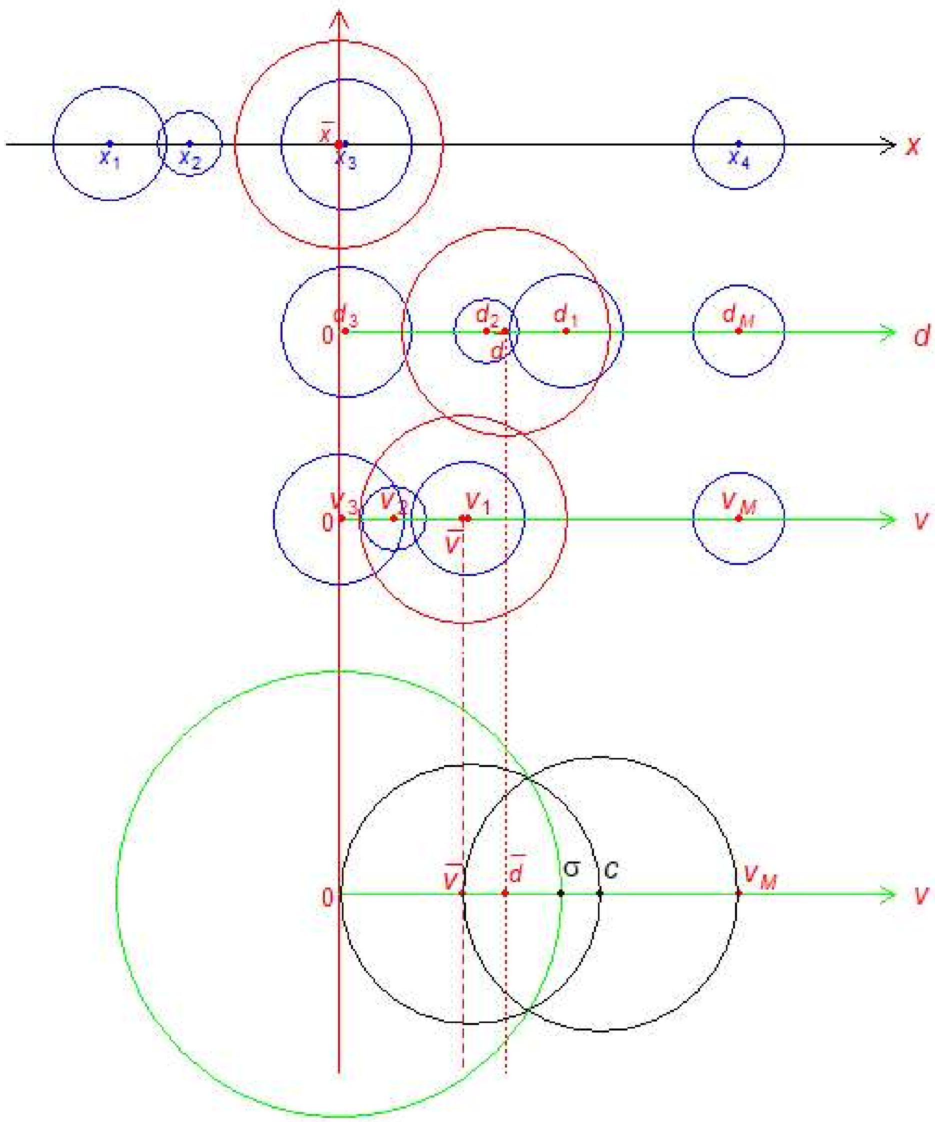


Figure 3: Visualizing the mean, MD, and SD in Example 1.

4. Conclusion

A disc plot is a convenient way to visualize the joint PMF of two DRVs as well as the PMF of a single DRV. Here, we have shown that a disc plot also yields the mean, MD, and SD of *X*, even when the values of x and p(x) are not disclosed! To save space, one may replace the circles used in all figures with the corresponding upper semicircles.

We hope that the exhibited visualizations will empower students and users of probability and statistics. Visualization of all relevant statistical concepts from a disc plot for two DRVs or from a scatter plot of a random sample of bivariate data is a work in progress.

Acknowledgement

The authors thank their colleagues for some discussions.

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