Journal of Probability and Statistical Science
24(1), 1-15 Feb. 2026

A Dynamic Autoregressive Time Series Regression Model for Precipitation Using Particulate Matter and Carbon Dioxide as Exogenous Variables

Oyindamola R. Oyebanjo
Department of Mathematics and Statistics
Abiola Ajimobi Technical University, Nigeria

Timothy O. Olatayo
Department of Statistics
Olabisi Onabanjo University, Nigeria

Abass I. Taiwo
Department of Statistics
Olabisi Onabanjo University, Nigeria

ABSTRACT

This paper presents the Dynamic Autoregressive Time Series Regression Model (DATSRM) to forecast time series data characterised by complex seasonal patterns and external influences. Even though common models like Autoregressive with Exogenous Variables (ARX), Autoregressive Integrated Moving Average with Exogenous Variables (ARIMAX), Seasonal ARIMAX (SARIMAX), and Lagged Time Series Regression (LTSR) are widely used in time series analysis, they often struggle to effectively manage complex seasonal changes and important outside factors. DATSRM was developed to overcome these limitations by using exogenous variables to account for external factors, like the environment; autoregressive components to capture linear dependencies; and Fourier components to characterise seasonal and cyclical patterns. The model's effectiveness was evaluated using Nigerian datasets that included annual precipitation and air pollution metrics (carbon dioxide and particulate matter) from 1990 to 2022. The model's parameters were determined using the Ordinary Least Squares method. The Durbin-Watson statistic, autocorrelation analysis, Akaike information criterion (AIC) and Bayesian information criterion (BIC), residual diagnostics were used to assess the model's performance. Coefficient of determination (R^2), adjusted coefficient of determination ($\bar{R}^2$) were used for the model accuracy. Mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE) were also used to evaluate the forecast's accuracy. The results obtained indicated that DATSRM outperformed ARX, ARIMAX and LTSR in terms of R^2 (0.8318, 0.8884, 0.8965) and $\bar{R}^2$ (0.7991, 0.8433, 0.8587). DATSRM achieved the lowest AIC (11.1585, 11.3381, 11.2088) and BIC (11.4306, 11.5101, 11.3309) values, indicating better model fit. The forecast evaluation of DATSRM has the smallest value when compared with ARX, ARIMAX and LTSR model, this indicates that DATSRM is a better model compared with existing fitted model. Therefore, DATSRM provides a more accurate and reliable approach to modelling and forecasting datasets characterized by seasonal and periodic behaviors influenced by external variables.

Keywords: Dynamic Autoregressive Time Series Regression, Autoregressive with Exogenous Variable, Lagged Time Series Regression, Autoregressive Integrated Moving Average with Exogenous Variable, Fourier Technique

- Received September 26, 2025, in final form February 2026.
- Oyindamola R. Oyebanjo (corresponding author) is affiliated with Department of Statistics, Faculty of Natural and Applied Sciences, Abiola Ajimobi, Technical University, Ibadan, Nigeria.
oyebanjooyindamola@gmail.com

1. Introduction

Time series data, consisting of sequential observations indexed chronologically, are crucial in disciplines such as finance, climatology, epidemiology, and engineering (Weerakody *et al.*, 2021; Pandit *et al.*, 2021). These datasets typically exhibit trends, seasonal variations, cyclical patterns, and erratic fluctuations (Velicer and Fava 2003; Al-Khasawneh *et al.*, 2024), necessitating sophisticated analytical models. Autoregressive (AR), moving average (MA), autoregressive moving average (ARMA), and autoregressive integrated moving average (ARIMA) are conventional techniques that have proven effective for time series analysis. However, their efficacy diminishes when the data exhibits dynamic periodicities or external influences.

More intricate models such as seasonal autoregressive integrated moving average (SARIMA), autoregressive integrated moving average with exogenous variables (ARIMAX), and seasonal autoregressive integrated moving average with exogenous variables (SARIMAX) have expanded upon ARIMA by incorporating seasonal and exogenous components (Alharbi and Csala 2022; Zhang 2003). However, they continue to operate under the assumption that seasonality and other factors remain constant. In reality, phenomena such as economic cycles, illness trends, and meteorological patterns generally do not adhere to these principles, they alter their cyclical behaviours and are influenced by external factors (Box and Tiao 1975; De Gooijer and Hyndman 2006).

This study employs a hybrid model that integrates Autoregressive components to address inherent linear dependencies, Fourier series to characterise dynamic periodic and seasonal variations, and Exogenous variables to incorporate external influences. The Dynamic Autoregressive Time Series Regression Model (DATSRM) is superior for predictions when seasonality is irregular and external influences are present. This work elaborates on the theoretical development of DATSRM, its practical testing, and its comparative analysis with existing models. Time series modelling has evolved significantly, progressing from basic linear trend analyses to intricate stochastic models. Individuals are increasingly focussing on Fourier-based methodologies due to their ability to encapsulate periodic variations through sinusoidal elements. Taiwo and Olatayo 2018 showed that Fourier regression is effective with hydrological and climatological datasets. Taiwo *et al.* 2019 developed a Fourier autoregressive model to analyse Nigeria's monthly precipitation data. The model was deemed sufficiently effective for modelling and predicting periodicity and seasonality in Nigerian rainfall time series data, as well as in other regions exhibiting analogous periodic fluctuations in rainfall data.

Awoyemi *et al.* (2024) introduced a model that combines Fourier regression with trend time series regression, utilised for Nigerian crude oil prices, macroeconomic indicators, and precipitation data. This hybrid method accurately simulated and predicted datasets with trend-cyclical fluctuations. Prahutama *et al.* (2018) previously employed Fourier regression utilising ordinary least squares and generalised cross-validation to analyse inflation in Indonesia's food sector. Extensions of ARIMA, like ARIMAX and SARIMAX, integrate external regressors, enhancing forecasts of electricity demand, economic indicators, and environmental variables (Bennett *et al.,* 2024). Olatayo and Ekerikevwe (2022) formulated a LOG-ARIMAX model to address heavy-tailed long-memory data, whereas Liu *et al.* (2023) utilised SARIMA to predict acute mountain sickness with considerable precision. Additional adaptations encompass Liu and Zhu (2018) improving ARX models in the context of errors-in-variables, and Kaur and Rakshit (2019) demonstrating that PAR models surpass SARIMA for rainfall predictions in Punjab. Applications encompass forecasting sugarcane production (Gopinath & Kavithamani, 2019), analysing exchange rate dynamics (Velicer & Fava, 2003), estimating insurance reserves (Ulyah *et al.,* 2019), assessing power consumption (Shadkam, 2020), and predicting rainfall in Australia (Warsono *et al.,* 2019). Although ARIMAX, SARIMAX, and ARX models exhibit extensive applicability, they frequently encounter difficulties with dynamic seasonal cycles and various external shocks.

The literature increasingly concurs that hybrid models integrating Fourier decomposition and regression analysis demonstrate superior performance. However, few studies have directly integrated Fourier analysis with autoregression while incorporating external factors. This discrepancy prompted the proposed model, which integrates multiple methodologies to enhance forecast accuracy and adaptability.

2. Methodology

The methodology of this study outlines the mathematical formulation and estimation techniques for the proposed Dynamic Autoregressive Time Series Regression Model (DATSRM). This model integrates three components: autoregressive (AR) terms for capturing historical dependencies, Fourier series for modelling seasonal-periodicity, and exogenous variables for external influences.

$$y_t = \phi_0 + \sum_{i=1}^{m} \phi_i y_{t-i} + \sum_{i=1}^{m} (c_i \sin \omega_i t + d_i cos\, \omega_i t) + \sum_{i=1}^{m} \Gamma_i x_{t-i} + \varepsilon_t \qquad (1)$$

where y_t is the dependent variable, x_{t-i} are the lagged values of the exogenous variable, Γ_i is the coefficients associated with the exogenous variable, ϕ_i are the autoregressive parameters, y_{t-i} are the lagged values of the time series variable, $c_i and\, d_i$ is the coefficients of the sinusoidal terms, ω_i is the angular frequency corresponding to the ith Fourier terms, t is the index time, m is the year index, $sin\omega\ and\ cos\omega$ is the trigonometric functions and ε_t is the error term.

The model was applied to Nigerian annual meteorological dataset namely precipitation and external variable are air pollution dataset namely particulate matter and carbon dioxide from 1990 to 2022. Both external variable where used individually and used together in the model. The model parameters are estimated using the Ordinary Least Squares (OLS) technique. Below are the estimated parameters for order 1:

$$\begin{bmatrix} \sum_{i=1}^{m} y_t \\ \sum_{i=1}^{m} y_t y_{t-1} \\ \sum_{i=1}^{m} y_t \, sin\omega t \\ \sum_{i=1}^{m} y_t \, cos\omega t \\ \sum_{i=1}^{m} y_t x_{t-1} \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^{m} y_{t-1} & \sum_{i=1}^{m} sin\omega t & \sum_{i=1}^{m} cos\omega t & \sum_{i=1}^{m} x_{t-1} \\ \sum_{i=1}^{m} y_{t-1} & \sum_{i=1}^{m} y_{t-1} y_{t-1} & \sum_{i=1}^{m} sin\omega t y_{t-1} & \sum_{i=1}^{m} cos\omega t y_{t-1} & \sum_{i=1}^{m} x_{t-1} y_{t-1} \\ \sum_{i=1}^{m} sin\omega t & \sum_{i=1}^{m} y_{t-1} sin\omega t & \sum_{i=1}^{m} sin\omega t sin\omega t & \sum_{i=1}^{m} cos\omega t sin\omega t & \sum_{i=1}^{m} x_{t-1} sin\omega t \\ \sum_{i=1}^{m} cos\omega t & \sum_{i=1}^{m} y_{t-1} cos\omega t & \sum_{i=1}^{m} sin\omega t cos\omega t & \sum_{i=1}^{m} cos\omega t cos\omega t & \sum_{i=1}^{m} x_{t-1} cos\omega t \\ \sum_{i=1}^{m} x_{t-1} & \sum_{i=1}^{m} y_{t-1} x_{t-1} & \sum_{i=1}^{m} sin\omega t x_{t-1} & \sum_{i=1}^{m} cos\omega t x_{t-1} & \sum_{i=1}^{m} x_{t-1} \, x_{t-1} \end{bmatrix} \begin{bmatrix} \phi_0 \\ \phi_1 \\ c_1 \\ d_1 \\ \Gamma_1 \end{bmatrix} \quad (2)$$

To detect the optimal model Akaike information criterion (AIC) and Bayesian information criterion (BIC) were used;

$$AIC = -2 \log l + 2\hat{S} \quad (3)$$

$$BIC = \hat{S} logm - 2logl \quad (4)$$

To assess model adequacy, the coefficient of determination (R^2) and adjusted R^2 are used;

$$R^2 = 1 - \frac{SSE}{SST} \quad (5)$$

$$\bar{R}^2 = 1 - \frac{m-1}{m-k}(1 - R^2) \quad (6)$$

The forecasting measures used in this study are mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE);

$$MAE = \frac{1}{h+1} \sum_{t=s}^{h+s} (\hat{X}_t - X_t)^2 \quad (7)$$

$$RMSE = \sqrt{\frac{1}{h+1}\sum_{t=s}^{h+s}(X_t)^2} \quad (8)$$

$$MAPE = \frac{100}{h+s}\sum_{t=s}^{h+s}\left|\frac{\hat{X}_t - X_t}{\hat{X}_t}\right| \quad (9)$$

To check the stability of error in this study, the Durbin-Watson statistic, autocorrelation and partial autocorrelation functions (ACF and PACF) were used.

$$ACF = \frac{cov(y_t, y_{t-i})}{\sqrt{vary_t vary_{t-i}}} \quad (10)$$

$$PACF = \frac{cov\left(T_t, \frac{T_{t-i}}{T_{t-i+1}}\right)}{\sqrt{var\left(\frac{T_t}{T_{t-i+1}}\right) var\left(\frac{T_{t-i}}{T_{t-i+1}}\right)}} \quad (11)$$

The model was compared with models like autoregressive integrated moving average with exogenous variable, autoregressive exogenous variable and lagged time series regression.

3. Results and Discussion

An initial exploration of Nigeria's annual climatic and air pollution data from 1990 to 2022 reveals non-stationary behaviours characterised by evident cyclical and seasonal fluctuations in precipitation, particulate matter. Given these dynamics, the application of the proposed Dynamic Autoregressive Time Series Regression Model (DATSRM), which accommodates both periodic-seasonality and exogenous influences, is methodologically justified.

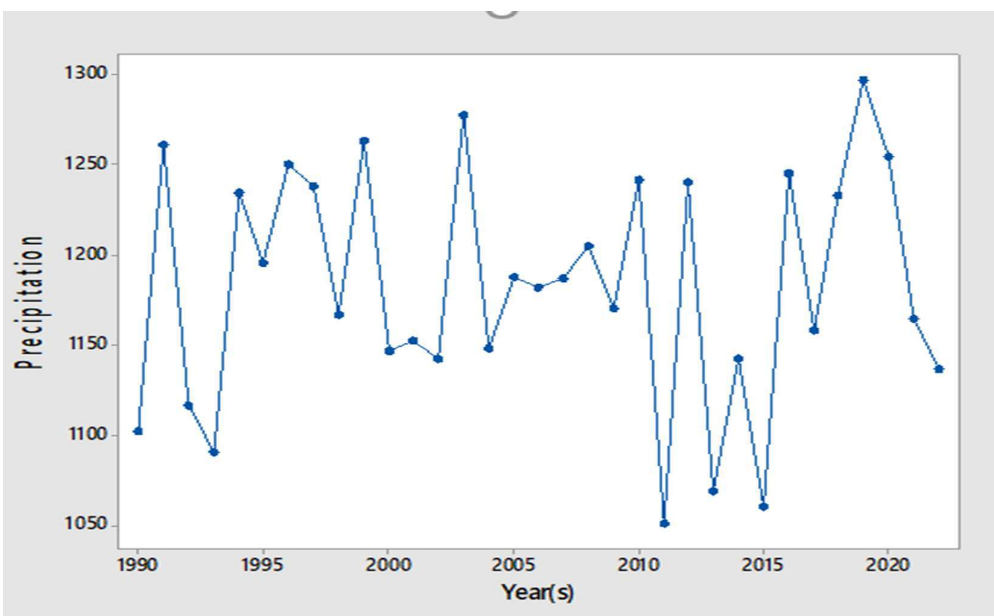

Figure 1: Time plot of Nigeria annual mean precipitation from 1990 – 2022

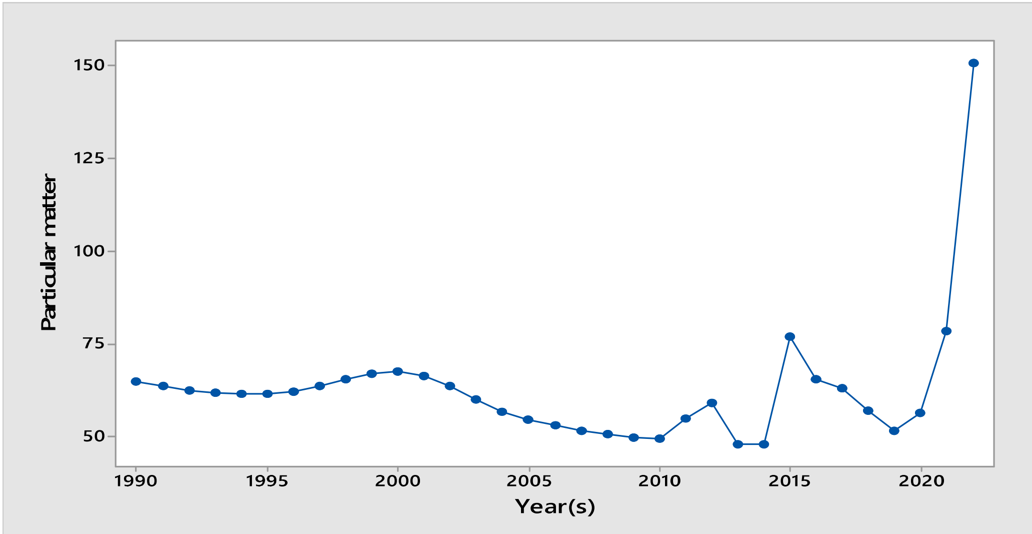

Figure 2: Time plot of Nigeria annual mean particulate matter from 1990 – 2022

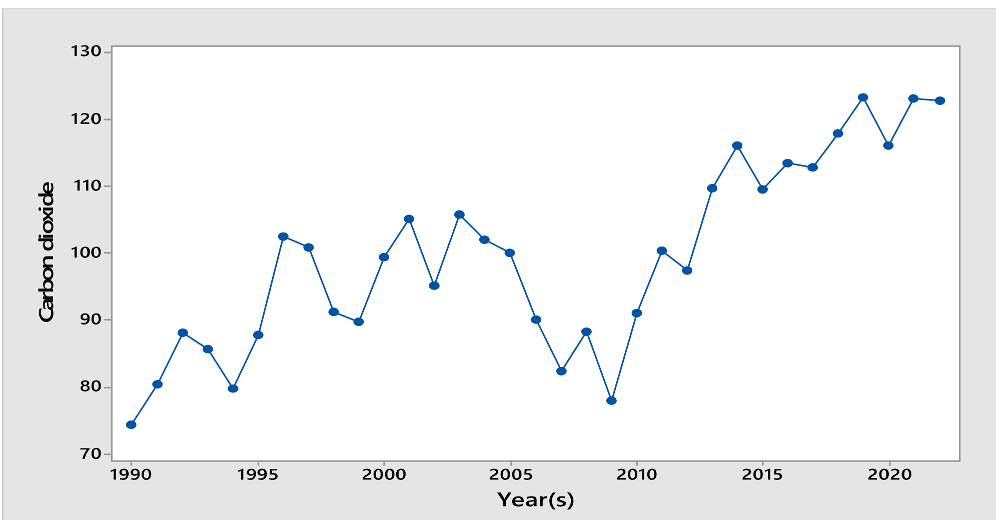

Figure 3: Time plot of Nigeria annual mean carbon dioxide from 1990 – 2022

Figure 1 provides a time plot visualisation of Nigeria's yearly precipitation from 1990 to 2022, indicating a series with non-constant mean and variance. A thorough examination indicated an annual cyclical-seasonal fluctuation. Figures 2 and 3, which show the annual mean particulate matter and carbon dioxide in Nigeria from 1990 to 2022, show cyclical-seasonal fluctuation, indicating that the dataset is non-stationary.

Using precipitation as the dependent variable and particulate matter as the exogenous regressor, the DATSRM was estimated via OLS.

The DATSR (1) model for the OLS technique is

$$y_t = 1714.764 - 0.2242y_{t-1} + 45.4617sin\frac{2\pi}{12}t - 45.9263\cos\frac{2\pi}{12}t - 4.1993x_{pt} - 0.2989x_{pt-1} \quad (12)$$

The model explained 83.18% of the variation in precipitation ($R^2 = 0.8318$), with an adjusted R^2 of 0.7991 and no significant autocorrelation (Durbin-Watson = 1.88). Fourier terms effectively captured periodicity, while particulate matter exerted a statistically significant negative effect on precipitation.

Table 1: Autocorrelation and Partial Autocorrelation Plot of the Residual for Dynamic Autoregressive Time Series Regression Model (Precipitation and Particulate Matter)

Autocorrelation	Partial Correlation		AC	PAC	Q-Sta...	Prob
		1	-0.19...	-0.19...	1.3085	0.253
		2	-0.03...	-0.07...	1.3640	0.506
		3	-0.07...	-0.10...	1.5961	0.660
		4	0.003	-0.03...	1.5965	0.809
		5	-0.18...	-0.21...	3.0155	0.698
		6	-0.10...	-0.22...	3.5057	0.743
		7	0.012	-0.12...	3.5122	0.834
		8	-0.04...	-0.17...	3.6051	0.891
		9	0.223	0.117	6.0072	0.739
		1...	-0.18...	-0.23...	7.7613	0.652
		1...	0.019	-0.18...	7.7799	0.733
		1...	0.082	-0.03...	8.1510	0.773
		1...	0.242	0.208	11.536	0.566
		1...	-0.08...	0.096	11.950	0.610
		1...	-0.08...	-0.07...	12.433	0.646
		1...	0.018	-0.04...	12.455	0.712

Table 1 reveals that the ACF and PACF plots of the residuals have no significant linear connections between lags.

Table 2: Values of Coefficient of Determination and Adjusted Coefficient of Determination for Dynamic Autoregressive Time Series Regression Model (Precipitation and Particulate Matter)

Model(s)	Coefficient of Determination (R^2)	Adj. Coefficient of Determination ($\bar{R}^2$)
LTSR	0.4346141	0.3887450
ARX	0.5011403	0.4754504
ARIMAX	0.5102555	0.4895354
DATSR	0.8317826	0.7991498

Table 2 shows that the R^2value for DATSR model is better than the compared conventional models considered also the value of $\bar{R}^2$signified that the DATSR model has better goodness of fit and predictive power for explaining the variations of Nigerian annual mean climatic change.

Table 3: Forecast Evaluation for Dynamic Autoregressive Time Series Regression Model (Precipitation and Particulate Matter)

Model(s)	MAE	RMSE	MAPE
DATSR	43.01461	53.43740	3.662724
ARX	51.16711	63.09906	4.366669
LTSR	51.33313	63.18878	4.382095
ARIMAX	57.97647	72.58668	4.877366

Table 4: Values of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for Dynamic Autoregressive Time Series Regression Model (Precipitation and Particulate Matter)

Model(s)	AIC	BIC
DATSR	11.15854	11.43063
LTSR	11.31195	11.44799
ARX	11.36945	11.55084
ARIMAX	11.60198	11.87680

The values of forecast evaluation metrics given in Table 3 indicated that the forecast evaluation of the DATSRM has lower values. Table 4 shows that DATSR has the lowest AIC and BIC values of any other model, indicating that it provides a better balance of accuracy and complexity. This makes the proposed model forecast has better accuracy than all other models considered.

Using precipitation as the dependent variable and carbon dioxide as the exogenous variable, the DATSRM was estimated via OLS.
The DATSR (1) model for the OLS technique is

$$y_t = 1357.172 - 0.1743y_{t-1} + 14.0634sin\frac{2\pi}{12}t - 33.9899\cos\frac{2\pi}{12}t - 1.9648x_{ct} + 2.2621x_{ct-1} \quad (13)$$

The model explained 88.84% of the variation in precipitation ($R^2 = 0.8884$), with an adjusted R^2 of 0.8433 and no significant autocorrelation (Durbin-Watson = 2.30). Fourier terms effectively captured periodicity, while particulate matter exerted a statistically significant negative effect on precipitation.

Table 5: Autocorrelation and Partial Autocorrelation Function Plot of the Residual for Dynamic Autoregressive Time Series Regression Model (Precipitation and Carbon Dioxide)

Autocorrelation	Partial Correlation		AC	PAC	Q-Sta...	Prob
		1	-0.16...	-0.16...	0.9532	0.329
		2	0.231	0.210	2.9349	0.231
		3	-0.19...	-0.14...	4.4091	0.221
		4	-0.09...	-0.19...	4.7517	0.314
		5	-0.31...	-0.31...	8.8754	0.114
		6	-0.13...	-0.24...	9.6277	0.141
		7	-0.03...	-0.05...	9.6931	0.207
		8	-0.11...	-0.24...	10.252	0.248
		9	0.146	-0.12...	11.283	0.257
		1...	-0.03...	-0.24...	11.360	0.330
		1...	0.207	-0.12...	13.613	0.255
		1...	0.067	-0.03...	13.863	0.310
		1...	0.181	0.011	15.752	0.263
		1...	-0.00...	-0.01...	15.755	0.329
		1...	-0.00...	-0.05...	15.755	0.399
		1...	-0.06...	0.030	16.002	0.453

The ACF and PACF plots of the residuals revealed no significant linear connections between lags.

Table 6: Values of Coefficient of Determination and Adjusted Coefficient of Determination for Dynamic Autoregressive Time Series Regression Model (Precipitation and Carbon Dioxide)

Model(s)	Coefficient of Determination (R^2)	Adj. Coefficient of Determination ($\bar{R}^2$)
LTSR	0.3509864	0.3156145
ARX	0.5957951	0.5339503
ARIMAX	0.7554914	0.7277013
DATSR	0.8883687	0.8432518

Table 6 shows that the R^2 value for the DATSR model is better than all other conventional models considered also the value of $\bar{R}^2$ signified that the DATSR model has better goodness of fit and predictive power for explaining the variations of Nigerian annual mean climatic change.

Table 7: Forecast Evaluation for Dynamic Autoregressive Time Series Regression Model (Precipitation and Carbon Dioxide)

Model(s)	MAE	RMSE	MAPE
DATSR	47.19681	50.45561	4.355159
ARX	51.19681	58.45561	4.355159
LTSR	53.96801	64.37914	4.61012
ARIMAX	56.23786	72.43760	4.74083

Table 8: Values of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for Dynamic Autoregressive Time Series Regression Model (Precipitation and Carbon Dioxide)

Model(s)	AIC	BIC
DATSR	11.33805	11.51014
LTSR	11.34927	11.58532
ARX	11.36010	11.54149
ARIMAX	11.66805	11.98868

DATSRM, autoregressive exogenous, lagged time series regression and autoregressive integrated moving average exogenous (ARIMAX) forecast were compared based on their forecast evaluation that is MAE, RMSE and MAPE. The values of forecast evaluation metrics given in Table 7 indicated that the forecast evaluation of the DATSR model has lower values. Table 8 shows that the dynamic autoregressive time series regression (DATSR) model is the better choice for modelling precipitation and carbon dioxide due to its lowest AIC and BIC values.

Using precipitation as the dependent variable and particulate matter, carbon dioxide as the external influences, the DATSRM was estimated via OLS.

The DATSR (1) model for the OLS technique is

$$y_t = 1820.627 - 0.3354y_{t-1} + 45.1231 sin\frac{2\pi}{12}t - 51.5899\cos\frac{2\pi}{12}t - 3.6141x_{pt} - 0.5624x_{pt-1} - 1.6729x_{ct} + 1.7278x_{ct-1} \quad (14)$$

The dynamic autoregressive time series regression analysis equation for OLS showed that the coefficients noted that for every increase in time, particulate matter and carbon dioxide, there will be a decrease in precipitation yearly. The model explained 89.65% of the variation in precipitation ($R^2 = 0.8965$), with an adjusted R^2 of 0.8587 and no significant autocorrelation (Durbin-Watson = 2.18). Fourier terms effectively captured periodicity, while particulate matter exerted a statistically significant negative effect on precipitation.

Table 9: Autocorrelation and Partial Autocorrelation Function Plot of the Residual for Dynamic Autoregressive Time Series Regression Model (Precipitation, Particulate Matter and Carbon Dioxide)

Autocorrelation	Partial Correlation		AC	PAC	Q-Sta...	Prob
		1	-0.09...	-0.09...	0.2979	0.585
		2	0.085	0.077	0.5655	0.754
		3	-0.09...	-0.08...	0.9267	0.819
		4	-0.07...	-0.09...	1.1420	0.888
		5	-0.23...	-0.24...	3.4302	0.634
		6	-0.16...	-0.22...	4.5255	0.606
		7	0.017	-0.01...	4.5379	0.716
		8	-0.13...	-0.19...	5.4013	0.714
		9	0.200	0.083	7.3274	0.603
		1...	-0.14...	-0.23...	8.4165	0.588
		1...	0.084	-0.12...	8.7912	0.641
		1...	0.127	0.105	9.6733	0.645
		1...	0.243	0.223	13.086	0.441
		1...	-0.03...	0.017	13.162	0.514
		1...	-0.05...	-0.11...	13.375	0.573
		1...	-0.03...	-0.08...	13.450	0.640

The autocorrelation function and the partial autocorrelation function of residuals are given in Table 9 where the residuals do not have usual structure and significant at $\alpha = 0.05$.

Table 10: Values of Coefficient of Determination and Adjusted Coefficient of Determination for Dynamic Autoregressive Time Series Regression Model (Precipitation, Particulate Matter and Carbon Dioxide)

Model(s)	Coefficient of Determination (R^2)	Adj. Coefficient of Determination ($\bar{R}^2$)
LTSR	0.425606	0.384159
ARIMAX	0.511960	0.476635
ARX	0.678666	0.598475
DATSR	0.896455	0.858663

Table 10 shows that the R^2value for DATSR model is better than all other conventional models considered also the value of $\bar{R}^2$signified that the DATSR model has better goodness of fit and predictive power for explaining the variations of Nigerian annual mean climatic change.

Table 11: Forecast Evaluation for Dynamic Autoregressive Time Series Regression Model (Precipitation, Particulate Matter and Carbon Dioxide)

Model(s)	MAE	RMSE	MAPE
DATSR	42.17981	51.57477	3.58851
LTSR	50.79883	62.85932	4.33985
ARX	59.22260	76.18730	5.05889
ARIMAX	59.52891	73.13675	4.99902

Table 12: Values of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for Dynamic Autoregressive Time Series Regression 2Model (Precipitation, Particulate Matter and Carbon Dioxide)

Model(s)	AIC	BIC
DATSR	11.20879	11.33086
LTSR	11.36210	11.54349
ARIMAX	11.70881	12.07524
ARX	11.80730	12.03404

The diagnostic study verified that the DATSR technique was appropriate for forecasting Nigerian precipitation based on MAE, RMSE, and MAPE. The study revealed that the DATSR model is more suitable than the other compared models based on the AIC and BIC. Model diagnostics confirmed its adequacy, with the residuals showing no significant autocorrelation. When benchmarked against ARIMAX, ARX, and lagged regression models, DATSRM outperformed all others across all validation metrics, highest R^2, and better forecast accuracy as seen in Tables 1 – 12.

4. Conclusion

A Dynamic Autoregressive Time Series Regression (DATSR) model was developed in this study to handle recurring periodic seasonal variations with exogenous variable in datasets. The model went through the following stages: identification, estimation, diagnostics, and forecasting. The model outperformed the compared conventional models in its analysis meteorological data from Nigeria.

References

[1]. Al-Khasawneh M. A., Raza A., Khan S. U. R., and Khan Z. (2024). Stock Market Trend Prediction Using Deep Learning Approach. *Computational Economics*. https://doi.org/10.1007/s10614-024-10714-1
[2]. Alharbi F. R. and Csala D. (2022). A Seasonal Autoregressive Integrated Moving Average with Exogenous Factors (SARIMAX) Forecasting Model-Based Time Series Approach. *Inventions*, vol. 7 (4), 94. https://doi.org/10.3390/inventions7040094
[3]. Anderson P. L., Tesfaye Y. G., and Meerschaert M. M. (2007). Fourier-PARMA Models and Their Application to River Flows. *Journal of Hydrologic Engineering*, 12(5), 462–472. https://doi.org/10.1061/(asce)1084-0699(2007)12:5(462)
[4]. Awoyemi S. O., Taiwo I. A., and Olatayo O. T. (2024). Trend-Fourier Time Series Regression Model for Secular-Cyclical Datasets. *African Journal of Mathematics and Statistics Studies*, 7(2), 69–78. https://doi.org/10.52589/ajmss-svx0bdpo
[5]. Bennett C., Stewart R., and Lu J. (2014). Autoregressive with Exogenous Variables and Neural Network Short-Term Load Forecast Models for Residential Low Voltage Distribution Networks. *Energies*, 7(5), 2938–2960. https://doi.org/10.3390/en7052938
[6]. Box G. E. P. and Tiao G. C. (1975). Intervention Analysis with Applications to Economic and Environmental Problems. *Journal of the American Statistical Association*, 70(349), 70–79. [Online]. Available: https://doi.org/10.1080/01621459.1975.10480264
[7]. Bruce H., Andrews M. D., Dean R. S., and Caroline C. (2013). Building ARIMA and ARIMAX Models for Predicting Long-Term Disability Benefit Application Rates in the Public/Private Sectors, *Society of Actuaries*.

[8]. Ch D., Jamshaid, and Rashid A. (2016). Forecasting Of Wheat Production: A Comparative Study of Pakistan and India. *International Journal of Advanced Research*, 4(12), 698–709. https://doi.org/10.21474/ijar01/2449

[9]. Chang A. L. S., Ramba H., Jaaffar A. K. M., Phin C. K., and Mun H. C. (2016). Effect of climate variables on cocoa black pod incidence in Sabah using ARIMAX model. *AIP Conference Proceedings*, 1739, 020077. https://doi.org/10.1063/1.4952557

[10]. Cools M., Moons E., and Wets G. (2009). Investigating the Variability in Daily Traffic Counts through Use of ARIMAX and SARIMAX Models. *Transportation Research Record: Journal of the Transportation Research Board*, 2136(1), 57–66. https://doi.org/10.3141/2136-07

[11]. Dai J. and Chen S. (2019). The Application of ARIMA Model in Forecasting Population Data. *Journal of Physics: Conference Series*, 1324(1), 012100. https://doi.org/10.1088/1742-6596/1324/1/012100

[12]. Dai J., Xiao Y., Sheng Q., Zhou J., Zhang Z., and Zhu F. (2024). Epidemiology and SARIMA Model of Deaths in a Tertiary Comprehensive Hospital in Hangzhou from 2015 to 2022. *BMC Public Health*, 24(1). https://doi.org/10.1186/s12889-024-20033-7

[13]. De Gooijer J. G. and Hyndman R. J. (2006). 25 Years of Time Series Forecasting. *International Journal of Forecasting*, 22(3), 443–473. https://doi.org/10.1016/j.ijforecast.2006.01.001

[14]. Duran-Hernandez C., Ledesma-Alonso R., and Etcheverry G. (2020). Using Autoregressive with Exogenous Input Models to Study Pulsatile Flows. *Applied Sciences*, 10(22), 8228. https://doi.org/10.3390/app10228228

[15]. Gopinath M. and Kavithamani M. (2019). Production of Sugarcane Forecasting using ARIMAX Model. *International Journal of Innovative Technology and Exploring Engineering*, 8(12S), 596–600. https://doi.org/10.35940/ijitee.l1147.10812s19

[16]. Islam F. and Imteaz M. A. (2020). Use of Teleconnections to Predict Western Australian Seasonal Rainfall Using ARIMAX Model. *Hydrology*, 7(3), 52. https://doi.org/10.3390/hydrology7030052

[17]. Kaur J., Parmar K. S., and Singh S. (2023). Autoregressive Models in Environmental Forecasting Time Series: A Theoretical and Application Review. *Environmental Science and Pollution Research*, 30(8), 19617–19641. https://doi.org/10.1007/s11356-023-25148-9

[18]. Kaur S. and Rakshit M. (2019). Seasonal and Periodic Autoregressive Time Series Models Used for Forecasting Analysis of Rainfall Data. *International Journal of Advanced Research in Engineering & Technology*, 10(1). https://doi.org/10.34218/ijaret.10.1.2019.023

[19]. Khashei M., Bijari M., and Hejazi S. R. (2012). Combining Seasonal ARIMA Models with Computational Intelligence Techniques for Time Series Forecasting. *Soft Computing*, 16(6), 1091–1105. https://doi.org/10.1007/s00500-012-0805-9

[20]. Kim S., Lee P., Lee M., Kim J., and Na W. (2022). Improved State-of-Health Prediction Based on Auto-Regressive Integrated Moving Average with Exogenous Variables Model in Overcoming Battery Degradation-Dependent Internal Parameter Variation. *Journal of Energy Storage*, 46, 103888. https://doi.org/10.1016/j.est.2021.103888

[21]. Kumari S. and Muthulakshmi P. (2024). SARIMA Model: An Efficient Machine Learning Technique for Weather Forecasting. *Procedia Computer Science*, 235, 656–670. https://doi.org/10.1016/j.procs.2024.04.064

[22]. Kumari N. R. V., Ramakrishna N. G., Panasa N. V., and Sreenivas N. A. (2019). An Application of ARIMAX Model for Forecasting of Castor Production in India. *Journal of Oilseeds Research*, 36(4). https://doi.org/10.56739/jor.v36i4.136704

[23]. Liu X., Ma B., Gao M., and Chen L. (2024). A Novel Time-Varying Grey Fourier Model for Variable Amplitude Seasonal Fluctuation Sequences. *Grey Systems: Theory and Application*, 14(3), 473–490. https://doi.org/10.1108/gs-10-2023-0101

[24]. Liu J., Yu F., and Song H. (2023). Application of SARIMA Model in Forecasting and Analysing Inpatient Cases of Acute Mountain Sickness. *BMC Public Health*, 23(1). https://doi.org/10.1186/s12889-023-14994-4

[25]. Liu X. and Zhu Y. (2018). ARX Model Estimation of Multivariable Errors-in-Variables Systems. *IFAC-Papers On Line*, 51(15), 874–879. https://doi.org/10.1016/j.ifacol.2018.09.111

[26]. Olatayo T. O. and Ekerikevwe K. (2022). Performance Measures for Evaluating the Accuracy of Time Series Hybrid Model Using High Frequency Data. *Britain International of Exact Sciences (BIoEx) Journal*, 4, 244–259. https://doi.org/10.33258/bioex.v4i3.760

[27]. Pandit P., Krishnamurthy K., and Bakshi B. (2021). Prediction of crop yield and pest-disease infestation. *Elsevier eBooks*, 375–393. https://doi.org/10.1016/b978-0-12-823694-9.00021-9

[28]. Prahutama A., Suparti N., and Utami T. W. (2018). Modelling Fourier Regression for Time Series Data—A Case Study: Modelling Inflation in Foods Sector in Indonesia. *Journal of Physics: Conference Series*, 974, 012067. https://doi.org/10.1088/1742-6596/974/1/012067

[29]. Shadkam A. (2020). Using SARIMAX to forecast electricity demand and consumption in university buildings. M.S. thesis, Univ. of British Columbia. https://open.library.ubc.ca/collections/ubctheses/24/items/1.0391009

[30]. Taiwo A. and Olatayo T. (2018). An Improved Procedure for Fourier Regression Analysis. *Anale. Seria Informatică*, 16, 108–112.

[31]. Taiwo A. I., Olatayo T. O., Adedotun A. F., and Adesanya K. K. (2019). Modelling and Forecasting Periodic Time Series Data with Fourier Autoregressive Model. *Iraqi Journal of Science*, 1367–1373. https://doi.org/10.24996/ijs.2019.60.6.20

[32]. Taiwo A., Olatayo T., and Agboluaje S. (2020). Time Series Model Building with Fourier Autoregressive Model. *South African Statistical Journal*, 54(2), 243–254. https://doi.org/10.37920/sasj.2020.54.2.8

[33]. Ulyah S. M., Mardianto M. F. F., and Sediono N. (2019). Comparing the Performance of Seasonal ARIMAX Model and Nonparametric Regression Model in Predicting Claim Reserve of Education Insurance. *Journal of Physics: Conference Series*, 1397(1), 012074. https://doi.org/10.1088/1742-6596/1397/1/012074

[34]. Velicer W. F. and Fava J. L. (2003). Time Series Analysis. *Handbook of Psychology*, vol. 2, J. Schinka and W. F. Velicer, Eds., I. B. Weiner, Ed.-in-Chief, New York: Wiley, 581–606.

[35]. Warsono W., Russels E., Wamiliana W., Widiarti W., and Usman M. (2019). Vector Autoregressive with Exogenous Variable Model and its Application in Modelling and Forecasting Energy Data: Case Study of PTBA and HRUM Energy. *International Journal of Energy Economics and Policy*, 9(2), 390–398. https://www.econjournals.com/index.php/ijeep/article/view/7223

[36]. Weerakody P. B., Wong K. W., Wang G., and Ela W. (2021). A Review of Irregular Time Series Data Handling with Gated Recurrent Neural Networks. *Neurocomputing*, 441, 161–178.

[37]. Zhang G. P. (2003). Time Series Forecasting Using a Hybrid ARIMA and Neural Network Model. *Neurocomputing*, 50, 159–175. https://doi.org/10.1016/S0925-2312(01)00702-0