Journal of Probability and Statistical Science
23(1), 1-15 Aug. 2025

A Sojourn with Numerical Approximations to the Normal Distribution Suitable for the Undergraduate Classroom

Edward W. Swim
Department of Mathematics and Statistics
Sam Houston State University
Huntsville, TX 77340

Stephen M. Scariano
Department of Mathematics and Statistics
Sam Houston State University
Huntsville, TX 77340

ABSTRACT
Several transcendental approximations to the Normal Distribution are contrasted relative to mean absolute as well as maximum absolute error. Low degree Lagrange and Chebyshev polynomial interpolants are similarly studied to reduce mathematical complexity. Ultimately, a piecewise polynomial approximation, easily understood and handy for students, is explored as an alternative. This approximation is amenable to simple programming on a graphing calculator. Classroom examples are proposed and discussed.

1. Introduction

One of the premier examples that students studying introductory mathematical probability and statistics encounter, for which the Fundamental Theorem of Calculus, (FTC) is ineffective for purposes of calculating closed-form probabilities, is the Normal Cumulative Distribution Function (NCDF),

$$\Phi(x)=\begin{cases}\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{x}e^{-\left(t^2/2\right)}dt, & -\infty<x<\infty\\[2ex] \dfrac{1}{2}+\dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{0}^{x}e^{-\left(t^2/2\right)}dt, & x\geq 0.\end{cases} \tag{1}$$

Students at this level are usually annoyed at being relegated to the use of numerical tables or software for approximating NCDF probabilities, particularly after learning of the astounding applicability of FTC in previous calculus courses. Clearly, the need to explore numerical approximations to NCDF in some detail with these students is both relevant and justified. On the other hand, introductory, applied, statistics courses, crafted primarily for students with minimal mathematics backgrounds, typically adapt readily to the use of NCDF tables or statistical software to aid computations. Yet, the risk of student distraction using tables or software may inhibit the essential insights behind the calculations to be performed, reducing the whole effort to an inert exercise from their viewpoint. Again, some reasonable numerical elaboration of NCDF, at the appropriate developmental level, is crucial, certainly for these students.

The title specifies the central theme of this article. In Section 2, we study some historically useful, and sometimes clever, transcendental approximations to NCDF readily found in the literature. Bear in mind that no attempt to survey or catalogue the plethora of NCDF approximations is ventured here. Our primary goal is to migrate from quite sophisticated

- Received July 23, 2025, in final form August 2025.
- Dr. Stephen M. Scariano (corresponding author) is affiliated with Department of Mathematics and Statistics, Sam Houston State University, Huntsville, TX 77340
sms049@shsu.edu

approximations and engender the development of (low degree) polynomial, or piecewise polynomial, NCDF approximations which are both reasonably accurate as well as insightfully accessible to students. In Section 3, we customize some of the voluminous contributions of Joseph-Louis Lagrange to produce four low degree, polynomial approximation to NCDF, together with their respective approximation errors, while Section 4 uses the contributions of Pafnuty Lvovich Chebyshev to similarly develop a cubic Chebyshev polynomial approximant to NCDF. Section 5 discusses Hoyt's piecewise polynomial approximant to NCDF, while in Section 6 we construct a straightforward piecewise polynomial approximation easily accessible for student use. Suitable classroom examples are offered in Section 7, followed by conclusions and recommendations in Section 8.

2. Some Historical Perspectives

Many attempts have been made to approximate NCDF in (1) using well-behaved polynomials both for educational purposes and for numerical efficiency. One notable difficulty is the inefficiency in using a straightforward Taylor series expansion. As shown by Marsaglia [7], such a strategy provides a great deal of accuracy but can require hundreds of terms to maintain acceptable precision for large values of $\boldsymbol{x}$. On the other hand, many clever strategies have been developed that provide a consistently reasonable degree of accuracy on a wide interval without requiring an extremely high degree of polynomial approximation. We first present a sample of approximations to $\Phi(x)$ which shall be used as a basis for comparison for later discussion.

Famously, in 1945 Polya [9] published the approximation

$$\text{Polya: } \hat{\Phi}(x) = \frac{1}{2}\left(1 + \sqrt{1 - e^{-\left(2x^2/\pi\right)}}\right) \tag{2}$$

in an effort to simplify calculations in mathematical physics. Equation (2) has become somewhat of a standard in the literature for approximating $\Phi(x)$. In 1951, Cadwell [3] proposed

$$\text{Cadwell: } \hat{\Phi}(x) = \frac{1}{2}\left(1 + \sqrt{1 - \exp\left[-\frac{2x^2}{\pi} + \frac{2}{3\pi^2\left(\pi - 3\right)x^4}\right]}\right), \tag{3}$$

extending Polya's approximation and reducing the maximum error by roughly 80% using a quartic in place of a quadratic rational function in the exponent.

Hart [5] compiled almost one hundred approximations to $\Phi(x)$ with varying degrees of complexity, precision, and accuracy. One such approximation is

$$\text{Hart: } \hat{\Phi}(x) = 1 - \frac{\varphi(x)}{x}\left[1 - \left(\frac{\sqrt{1 + 0.282455x^2}}{\left(1 + 0.212024x^2\right)}\right)\left(x\sqrt{\frac{\pi}{2}} + \sqrt{\frac{1}{2}\pi x^2 + \frac{\sqrt{1 + 0.282455x^2}}{\left(1 + 0.212024x^2\right)}\exp(-x^2/2)}\right)^{-1}\right], \tag{4}$$

where $\varphi(x) = (2\pi)^{-1}\exp(-x^2/2)$ represents the standard normal density function.

Several examples in the literature embed a high-degree polynomial within an algebraic formula to achieve high levels of approximation accuracy. For example, Zelen and Severo [10] construct a highly accurate algorithm,

$$\text{Zelen and Severo: } \hat{\Phi}(x) = 1 - \varphi(x)\left(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5\right), \tag{5}$$

where $t = (1 + 0.2316419\boldsymbol{x})^{-1}$, $b_1 = 0.31938153$, $b_2 = -0.35656378$, $b_3 = 1.78147794$, $b_4 = 1.82125598$, and $b_5 = 1.33027443$. This approximation produces a maximum error that is much smaller than previous approximations considered here. These authors noted that $\varphi(x)$ itself can also be approximated using polynomials to avoid any evaluation of transcendental functions, and this results in only a modest increase in error.

In 1975, Carta [4] developed a table of approximations using rational functions. A commonly used example is

$$\text{Carta: } \hat{\Phi}(x) = 1 - 0.5\left(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5\right)^{(-16)}, \tag{6}$$

where a0 = 0.9999998582, a1 = 0.0487385796, a2 = 0.02109811045, a3 = 0.003372948927, a4 = 0.00005172897742, and a5 = 0.0000856957942. This approximation produces error values that are similar to those achieved with the Polya approximation. Seeking to build a simple expression that compared favorably with the Hart approximation in equation (4), Byrc [2] constructed rational functions, including

$$\text{Byrc: } \widehat{\Phi}(x) = 1 - \frac{x^2 + 5.575192695x + 12.77436324}{x^3\sqrt{2\pi} + 14.38718147x^2 + 31.53531977x + 25.548726} e^{-\left(\frac{x^2}{2}\right)}. \tag{7}$$

As illustrated in Tables 1 and 2, Byrc's approximation compares favorably with the work of Zelen and Severo in equation (5), and these two approaches provide the best overall results among the examples considered here. Table 1 provides the maximum absolute error for each approximation on the intervals $[0,1]$; $[1,3]$; and $[3,4]$. Table 2 presents the mean absolute error for each approximation over the interval $[0,4]$, since the CDF $\Phi(x)$ is very close to the value 1.0 for $x > 4$.

Table 1: Maximum Absolute Errors for Some Common Approximations of $\Phi(x)$.

	Range of the Standard Normal Variable		
Approximation Method	0 – 1.0	1.0 – 3.0	3.0 – 4.0
Polya (1945)	1.774×10^{-3}	3.146×10^{-3}	5.371×10^{-4}
Cadwell (1951)	8.323×10^{-5}	6.684×10^{-4}	4.155×10^{-4}
Hart (1968)	5.808×10^{-5}	5.790×10^{-5}	2.687×10^{-7}
Zelen and Severo (1964)	1.120×10^{-5}	1.095×10^{-5}	4.990×10^{-6}
Carta (1975)	2.868×10^{-3}	2.457×10^{-3}	7.160×10^{-5}
Byrc (2001)	1.185×10^{-5}	1.873×10^{-5}	2.051×10^{-6}

Table 2: Mean Absolute Errors on $[0,4]$ for Some Common Approximation of $\Phi(x)$.

Approximation Method	Mean Absolute Error
Polya (1945)	7.865×10^{-4}
Cadwell (1951)	1.671×10^{-4}
Hart (1968)	1.492×10^{-5}
Zelen and Severo (1964)	5.980×10^{-6}
Carta (1975)	7.171×10^{-4}
Byrc (2001)	6.921×10^{-6}

Many other recent approximations to $\Phi(x)$ are available, some of which often require the use of somewhat complicated transcendental functions. Still others may be expressed in a fairly simple format, such as the compact approximation

$$\text{Page}: \hat{\Phi}(x) = \left(1 + e^{-2t}\right)^{-1}, \quad \text{where } t = 0.7988x\left(1 + 0.04417x^2\right) \tag{8}$$

offered by Page [8]. Unfortunately, these typically lack the accuracy we seek.

All the approximations presented in this section rely on the use of square roots and the evaluation of exponential functions to achieve very accurate results. For students lacking a deep background in calculus, polynomial approximations can provide a simple approach where minimal knowledge of differentiation can be used to convert between a CDF and its corresponding PDF. In the following section, we illustrate effective methods for building such approximations with the goal, in general, of enhancing student insight into the ideas and methods for constructing useful approximations.

3. Lagrange Approximations to Φ(x)

Our first approach is to construct effective polynomial approximations using the standard Lagrange form for polynomial interpolation. In an effort to keep the degree of interpolant as low as possible, we choose nodes based on a uniform discretization of the range of the CDF $\Phi(x)$ rather than for the domain. For example, to construct a cubic interpolant first note that

$$\Phi(0) = 0.5, \quad \Phi(0.4307273) = 0.666667, \text{ and } \Phi(0.9674216) = 0.8333333.$$

Combining these three nodes with a fourth at either (3, 1) or (3.5, 1) produces approximations

$$P_3(x) = 0.5 + 0.41957x - 0.07432x^2 - 0.00333x^3. \tag{9}$$

and

$$P_{3.5}(x) = 0.5 + 0.42087x - 0.07867x^2 - 0.00022x^3. \tag{10}$$

Figure 1 illustrates the relationship between $P_3(x)$, $P_{3.5}(x)$ and $\Phi(x)$. Assuredly, both polynomials provide an overestimate in their respective tails. In addition to the inherent error associated with using a cubic polynomial interpolant over a wide interval, the fact that these polynomials overestimate $\Phi(x)$ near the tail effectively prevents them from being used as CDFs.

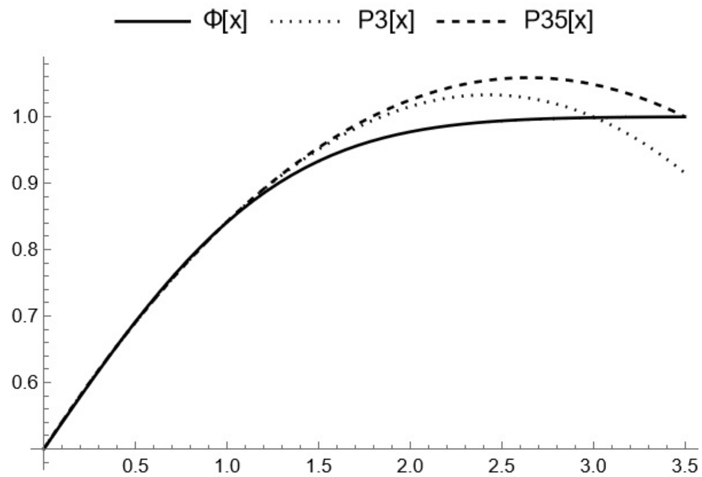

Figure 1: CDF $\Phi(x)$ (solid curve), along with interpolants $P_3(x)$ (dotted curve) with right endpoint (3, 1) and $P_{3.5}(x)$ (dashed curve) with right endpoint (3.5, 1).

To reduce the error in approximation seen in Figure 1, quintic interpolants are derived in a similar manner. Quartic polynomials are intentionally avoided to maintain a shape that closely matches that of $\Phi(x)$ on the interval $[0, 4]$. Following our previous procedure, five equally spaced nodes in the range are combined with either (3, 1) or (3.5, 1) from the tail to generate the following quintic approximations:

$$Q_3(x) = 0.5 + 0.397993x + 0.00779150x^2 - 0.0882330x^3 + 0.0257886x^4 - 0.00193701x^5, \tag{11}$$

$$Q_{3.5}(x) = 0.5 + 0.397952x + 0.00811747x^2 - 0.0890655x^3 + 0.0266322x^4 - 0.00222781x^5. \tag{12}$$

In Figure 2, we illustrate the relationships between $Q_3(x)$, $Q_{3.5}(x)$ and $\Phi(x)$. Both of our quintic interpolants have function values that are less than that of $\Phi(x)$, and both are much better approximations than their cubic counterparts, but both functions suffer on an interval where they are decreasing. Again, this prevents them from being used as true CDFs.

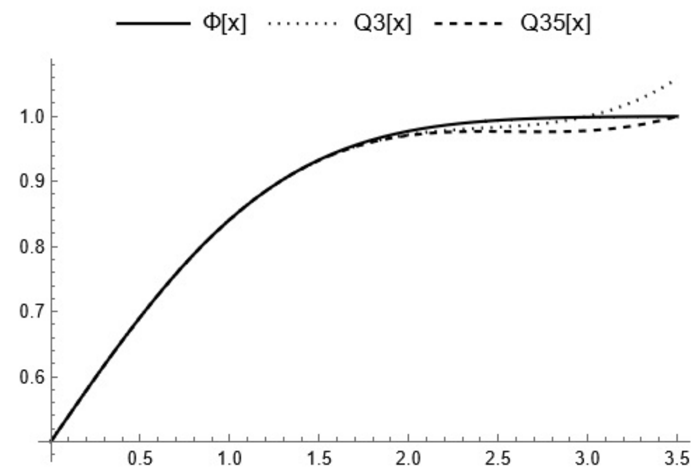

Figure 2: CDF $\Phi(x)$ (solid curve), along with interpolants $Q_3(x)$ (dotted curve) with right endpoint (3, 1), and $Q_{3.5}(x)$ (dashed curve) with right endpoint (3.5, 1).

Table 3: Maximum Absolute Errors for Lagrange Form Polynomial Approximations of $\Phi(x)$.

	Range of the Standard Normal Variable		
Polynomial Approximation	0 – 1.0	1.0 – 3.0	3.0 – 4.0
$P_3(x)$	1.919×10^{-3}	4.211×10^{-2}	2.239×10^{-1}
$P_{3.5}(x)$	2.079×10^{-3}	6.341×10^{-2}	8.929×10^{-2}
$Q_3(x)$	5.834×10^{-5}	1.056×10^{-2}	1.881×10^{-1}
$Q_{3.5}(x)$	6.361×10^{-5}	2.136×10^{-2}	5.809×10^{-2}

Table 4: Mean Absolute Errors on [0,4] for Lagrange Form Polynomial Approximations

Polynomial Approximation	Mean Absolute Error
$P_3(x)$	5.598×10^{-2}
$P_{3.5}(x)$	2.239×10^{-2}
$Q_3(x)$	4.703×10^{-2}
$Q_{3.5}(x)$	1.452×10^{-2}

Tables 3 and 4 give the errors for each of the approximations. These polynomial approximations maintain reasonable accuracy, but as noted before, none meets the requirements for a CDF on its domain of interpolation. As mentioned in [1] and elsewhere, increasing the degree of polynomial approximation will result in unsatisfactory results near the endpoints used for interpolation. In subsequent sections, we examine some well-known alternative approaches to building polynomial approximations.

4. Chebyshev Approximations to Φ(*x*).

An alternative to uniformly spaced nodes that produces a near minimax approximation (i.e., a nearly minimum value for error in the sense of the uniform norm) can instead be found using the Chebyshev nodes,

$$x_k = -\cos\left(\frac{k\pi}{n}\right), \quad k = 0,1,2,.......,n,$$

translated to the desired interval of interpolation. Unlike the standard Lagrange approach, where one can ensure that an interpolant achieves a right-hand limit of 0.5 at $x = 0$ and a left-hand limit of 1.0 for a particular abscissa x, a complicating factor is that no such guarantee can easily be made when using the Chebyshev nodes. However, it is relatively easy to instead construct a cubic polynomial interpolant using four Chebyshev nodes. In fact,

$$\hat{\Phi}(x) = T_3(x) = 0.491937 + 0.486062x - 0.153919x^2 + 0.01606304x^3, \qquad (13)$$

has mean absolute error of 1.841×10^{-3} on [0, 4], and its close fit to the CDF $\Phi(x)$ is shown in Figure 3.

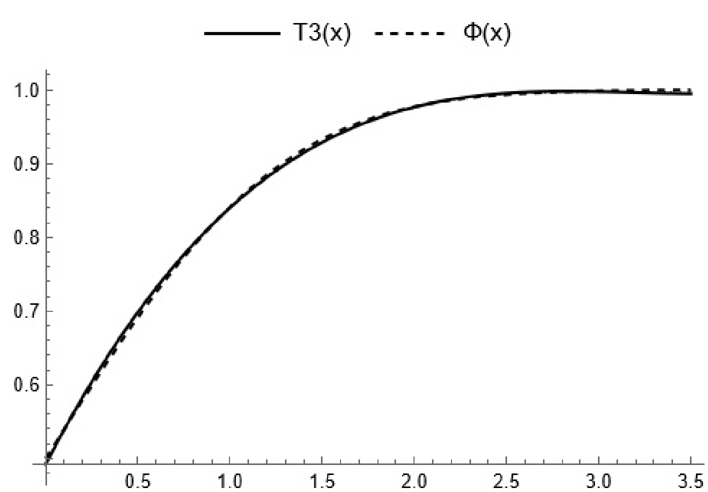

Figure 3: CDF $\Phi(x)$ (dashed curve) and cubic Chebyshev interpolant (solid curve)

Error values can be improved with a moderate increase in polynomial degree using this method, but convergence is slow and relies on a high degree of precision for the coefficients. Yet, the intended benefit of this approach is a single polynomial that is easy to evaluate and invert, and $T_3(x)$ is such a function. Due to numerical instability produced by even Chebyshev polynomial interpolations of increasing degree on a finite interval, we do not pursue that approach further. A common remedy for this problem is the use of piecewise polynomial approximations of low degree, as shown next.

5. Hoyt's Piecewise Polynomial Approximations to Φ(*x*).

Our stated goal is to construct an approximation that is easily understood and manipulated by students in introductory-level probability and statistics courses, which still preserves a high degree of numerical accuracy. For example, in 1968, Hoyt [6] suggested the quadratic, piecewise polynomial

$$\text{Hoyt PDF Approximation}: \; g(x) = \frac{d\hat{\Phi}(x)}{dx} = \begin{cases} (3+x)^2/16, & -3 \le x \le -1 \\ (3-x^2)/8, & -1 \le x < +1 \\ (3-x)^2/16, & +1 \le x \le +3 \\ 0, & \text{otherwise} \end{cases}, \tag{14}$$

This simple approximation is easily manipulated because straightforward integration results in the cubic, piecewise polynomial approximation

$$\text{Hoyt CDF Approximation}: \; G(x) = \begin{cases} \frac{1}{2} + \frac{3x}{8} - \frac{x^3}{24} = \frac{24+18x-2x^3}{48}, & 0 \le x < 1 \\ 1 - \frac{(3-x)^3}{48} = \frac{21+27x-9x^2+x^3}{48}, & 1 \le x < 3 \\ 1, & 3 \le x \end{cases} \tag{15}$$

Figure 4 clearly illustrates that G(*x*) closely follows the trajectory of Φ(*x*). However, it has significantly more error than most of the transcendental approximations already discussed. For example, the mean absolute error on $[0, 4]$ is 2.463×10^{-3}, similar to that of our Chebyshev approximation from Section 4, yet worse than the historical examples already considered in Section 2. In the next section, we will construct examples akin to the Hoyt approximation but built via interpolation of $\Phi(x)$ instead of its corresponding PDF.

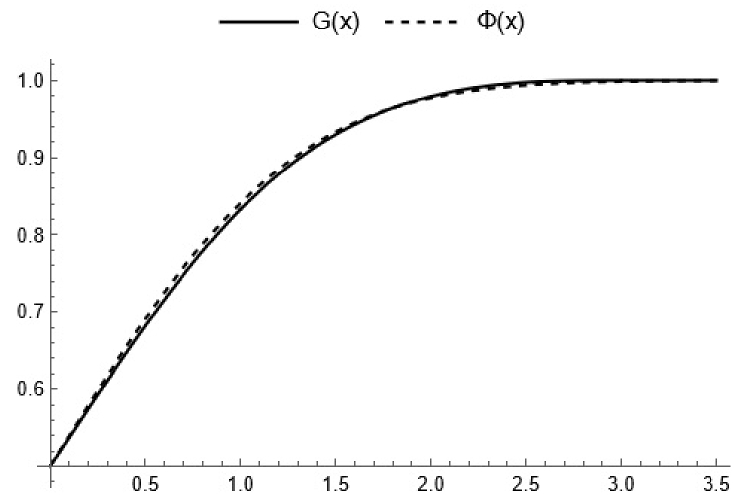

Figure 4: CDF $\Phi(x)$ (dashed curve) and the Hoyt approximation $G(x)$.

6. Developing an Accessible Piecewise Polynomial for Students

When teaching undergraduates ideas concerning continuous distributions, a typical textbook approach is to define the uniform distribution and illustrate its essential aspects, such as expected value, variance and standard deviation, using a few examples that include piecewise-defined functions, and then pivot swiftly to a discussion of the normal distribution. Our aim is to provide examples that help illustrate the value of so-called "bell shaped" probability distributions before introducing the standard normal distribution. A typical example is the piecewise linear PDF, commonly called the "tent" density, h(x), and its corresponding CDF, H(x), both shown in Figure 5. These functions are simple enough that elementary probability calculations, using area and function evaluation, are easily performed. However, H(x) is readily seen to be a very poor approximation to $\Phi(x)$, having mean absolute error 1.627×10^{-2} on the interval $[0, 4]$.

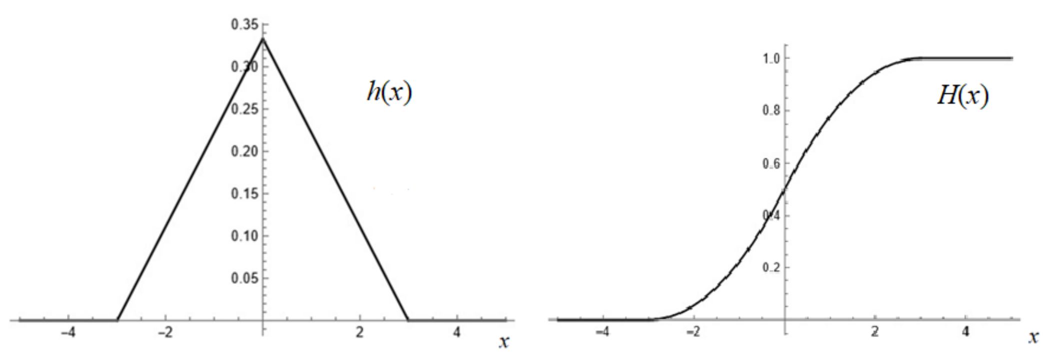

Figure 5. Tent PDF $h(x)$ and Tent CDF $H(x)$.

As a first step towards the goal of building a function in a similar format that can serve as an accurate approximation to $\Phi(x)$, modify $T_3(x)$ by constructing a simple piecewise cubic interpolant that attains the value of 0.5 at $x = 0.0166763$ and attains the value of 1.0 at $x = 4.022883$:

$$C_3(x) = \hat{\Phi}(x) = \begin{cases} 0.5, \; 0 \le x < 0.0166763 \\ .491937 + 0.486062x - 0.153919x^2 + 0.0160304x^3, \quad 0.0166763 < x < 4.022883 \\ 1, \quad 4.022883 \le x \end{cases} \tag{16}$$

Figure 6 illustrates the close fit of this approximation to $\Phi(x)$, like that observed in Figure 3.

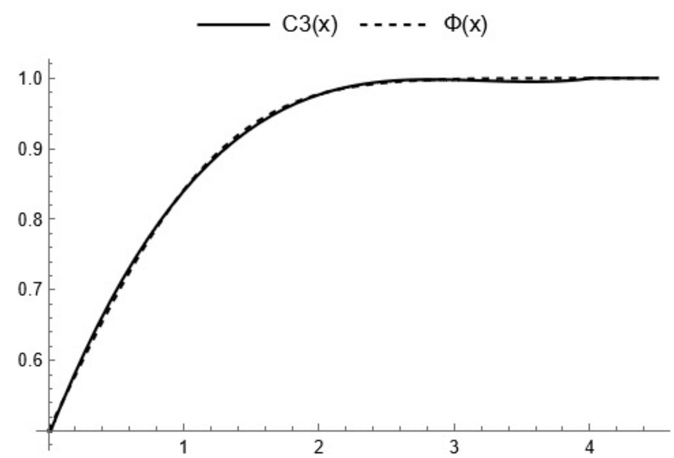

Figure 6: $C_3(x)$, a piecewise cubic CDF, in comparison with $\Phi(x)$.

Note that this approximation, C3(x), almost meets the definition for a CDF, since it is piecewise continuous, but it unfortunately decreases near the tail. To remedy this deficiency, we consider a piecewise cubic that is similar to C3(x), but still easier for students to manipulate:

$$S_3(x) = \hat{\Phi}(x) = \begin{cases} 0.5 + 0.48x - 0.15x^2 + 0.015x^3, \quad 0 < x < 8/3 \\ 0.46 + 0.54x - 0.18x^2 + 0.02x^3, \quad 8/3 < x < 3 \\ 1 \qquad , \; 3 < x \end{cases} \tag{17}$$

This approximation was constructed by adjusting the piecewise cubic interpolant $C_3(x)$ by insisting on nodes at $x = 0$, $x = 8/3$, and $x = 3$, which are specifically chosen to ensure that each piece is continuously increasing. The close approximation of $S_3(x)$ to $\Phi(x)$ is illustrated in Figure 7.

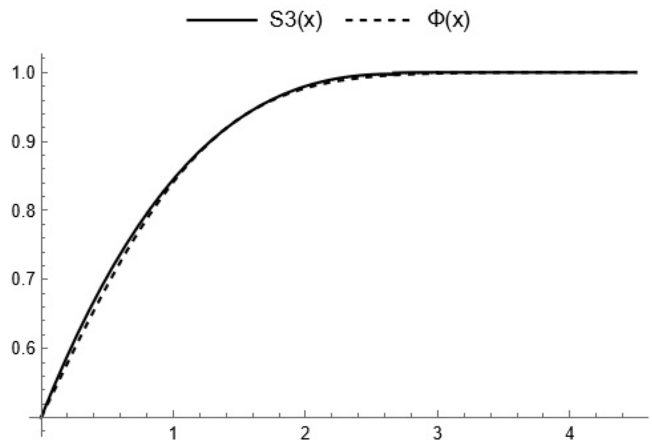

Figure 7: S_3, a piecewise cubic CDF, relative to $\Phi(x)$

Note that $S_3(x)$ has only one point of discontinuity, at x = 8/3, but reaches ordinate value 1 much faster than other approximations considered here. Table 5 and Table 6 present the error for this function, the Hoyt approximation $G(x)$ and the Chebyshev approximation $T_3(x)$, following the same format as before.

Table 5: Maximum Absolute Errors for Piecewise Polynomial Approximations of $\Phi(x)$

	Range of the Standard Normal Variable		
Approximation	0 – 1.0	1.0 – 3.0	3.0 – 4.0
$T_3(x)$	7.362×10^{-3}	4.516×10^{-3}	4.966×10^{-3}
$G(x)$	9.853×10^{-3}	8.011×10^{-3}	1.350×10^{-3}
$S_3(x)$	1.354×10^{-2}	3.730×10^{-3}	9.986×10^{-1}

Table 6: Mean Absolute Errors on $[0,4]$ for Piecewise Polynomial Approximations

Approximation	Mean Absolute Error
$T_3(x)$	1.841×10^{-3}
$G(x)$	2.463×10^{-3}
$S_3(x)$	3.386×10^{-3}

These results demonstrate that function $T_3(x)$, produced using Chebyshev interpolation, has error that is comparable with the historical approximations by Polya, Carta, and others. Except for values in the tail, i.e., for $x \in (3, 4)$, $T_3(x)$ also performs better than the Hoyt approximation. The error values for $S_3(x)$ are lowest for $x \in (1, 3)$, but higher than $T_3(x)$ and the approximation of Hoyt elsewhere. Based on the mean absolute errors presented in Table 6, we view both $T_3(x)$ and $S_3(x)$ (along with the approximation by Hoyt) as meeting our goals for accuracy and easy manipulation by students. In the next section, we illustrate how our approximations can be effectively used in an introductory level statistics course.

7. Classroom Examples

The following examples illustrate how our cubic Chebyshev interpolant $T_3(x)$ in equation (13) and the piecewise polynomial $S_3(x)$ in equation (17) can be used effectively.

Often, we ask students to use the CDF of a continuous distribution to calculate probabilities for various interesting events. Here, we are mindful that each calculation should be a reasonable approximation to the same probability calculation using the standard normal distribution. Note that if Z is a standard normal random variable, then

$$P(Z \le 1.25) = \Phi(1.25) \approx 0.8944. \tag{18}$$

With our cubic Chebyshev interpolant, we find that

$$T_3(1.25) = 0.491937 + 0.486062(1.25) - 0.153919(1.25)^2 + 0.0160304(1.25)^3 \approx 0.8903, \tag{19}$$

an approximation with 0.45% relative error. With our piecewise-cubic,

$$S_3(1.25) = 0.5 + 0.48(1.25) - 0.15(1.25)^2 + 0.015(1.25)^3 \approx 0.8949, \tag{20}$$

an approximation with 0.058% relative error, and one that is extremely simple for students to calculate as well as program into many common hand-held calculators.

Another common exercise for students is, given a CDF, obtain a formula for the corresponding PDF. For $x > 0$ and the cubic Chebyshev interpolant $T_3(x)$, this is easily accomplished:

$$t_3(x) = \frac{dT_3(x)}{dx} = 0.486062 - 2(0.153919)x + 3(0.0160304)x^2 = 0.486062 - 0.307838x + 0.0480912x^2. \tag{21}$$

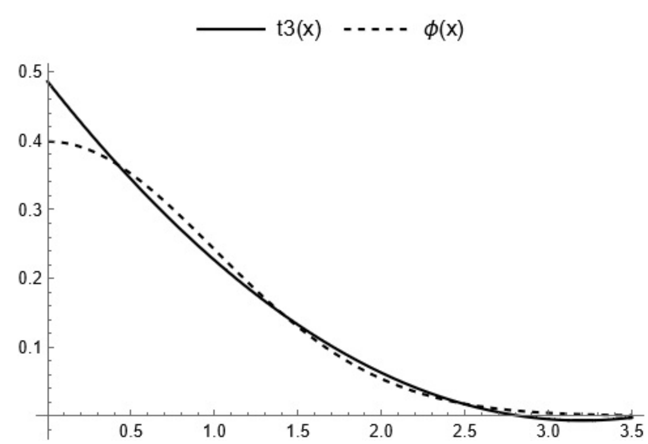

Figure 8: The graph of $t_3(x)$ in comparison with the standard normal PDF.

In Figure 8 we illustrate this approximation to the standard normal PDF. Although the fit is reasonably close, we note that t_3 cannot be a PDF since it takes on negative values near $x = 3$. In fact, our piecewise polynomial $S_3(x)$ performs somewhat better than the Chebyshev interpolant $T_3(x)$.

For $x > 0$, we have

$$s_3(x) = \frac{dS_3(x)}{dx} = \begin{cases} 0.48 - 0.3x + 0.045x^2, & 0 < x \le 8/3 \\ 0.54 - 0.36x + 0.08x^2, & 8/3 < x \le 3 \\ 1, & 3 < x \end{cases} \tag{22}$$

Figure 9 depicts this approximation to the NPDF. Notice that although small discontinuities are present, the function remains nonnegative on its domain.

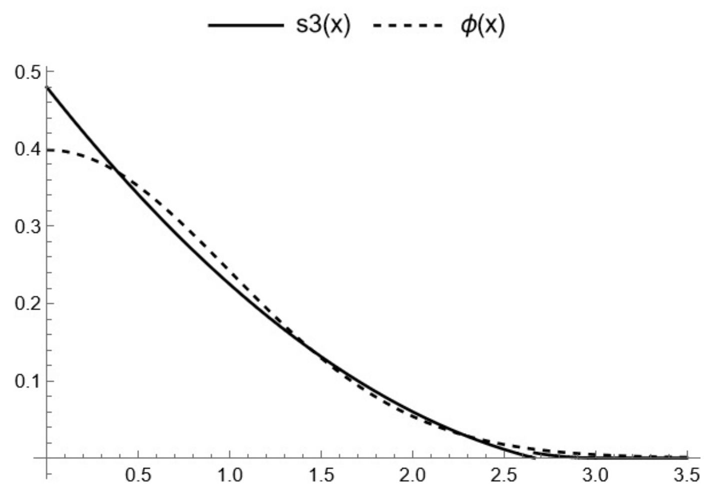

Figure 9: The graph of $\hat{p}_3(x)$ in comparison with the standard normal PDF

8. Conclusions

Numerous approximations to the standard normal cumulative distribution function are found in the literature. Unfortunately, much less effort has been directed to motivating student interest in these necessary approximations. The persistent pursuit of simple approximations for this distribution function, for the benefit of student insight, is the essence of this article. Here, the contextual discussion sequentially follows three themes: (i) the need for approximation, given the formidable form of the standard normal cumulative distribution function, (ii) a brief survey of some common, useful approximations, and (iii) motivational insight that students can grasp for simple, piecewise polynomial approximation, depending on the developmental level of the audience. That is, for students studying statistical methods in service courses, supporting the simple polynomial approximations studied here with clear tables and graphs is sufficient. Yet, students majoring in statistics, mathematics and the hard sciences can be challenged to gain appreciation for the deeper aspects of numerical approximation put forward by titans such as Polya, Lagrange, Chebyshev and others.

References

[1]. A. S. Ackleh, E. J. Allen, R. B. Hearfott and P. Seshaiyer. (2010). *Classical and Modern Numerical Analysis*. Chapman & Hall.

[2]. W. Byrc, A uniform approximation to the right normal tail integral, *App. Math. & Comp.* **127** (2001) pp.365—374.

[3]. J. H. Cadwell. (1951). The bivariate normal integral. *Biometrika*, 39, 475–479.

[4]. G. D. Carta. (1975). Low-order approximations for the normal probability integral and the error function, *Math. Comp.*, 29, 856–862.

[5]. J. F. Hart.(1968). *Computer Approximations*. Wiley.

[6]. J. P. Hoyt.(1968). A simple approximation to the standard normal probability density func- tion. *The American Statistician*, 22(2), 25–26.

[7]. G. Marsaglia.(2004). Evaluating the normal distribution. *J. Stat. Software*, 11.

[8]. E. Page(1977). Approximations to the cumulative normal function and its inverse for use on a pocket calculator. *Applied Statistics*, 26, 75–76.

[9]. G. Pólya(1945).Remarks on computing the probability integral in one and two dimensions. *Proc. of the First Berkeley Symposium on Mathematical Statistics and Probability*, 63–78.

[10]. M. Zelen and N. C. Severo, (1964). Probability functions. *Handbook of Mathematical Func- tions with Formulas, Graphs, and Mathematical Tables, eds. M. Abramowitz and I.A. Stegun*. National Bureau of Standards.