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An Autoregressive Process with Fourier Terms for Seasonal-Periodic Time Series Datasets

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ABSTRACT

A healthy economy depends on rapid economic growth but this can be seriously hampered by an unstable inflation rate. Consequently, the purpose of this study is to model and forecast Nigeria's yearly inflation rate by taking cognisance of the variation the series exhibited. The methods used are descriptive statistics, Fourier Autoregressive (FAR), Autoregressive Integrated Moving Average (ARIMA) and Seasonal ARIMA processes. Descriptive statistics outcomes of the series indicated that the mean is 15.85 with a standard deviation of 15.03. The time plot showed inflation rate series is non-stationary and exhibited seasonal and cyclical variations. The series is stationary during the initial difference, as demonstrated by applying the Augmented Dickey-Fuller test. The tentative FAR, ARIMA and SARIMA models were determined using autocorrelation and partial autocorrelation functions. The models estimated were chosen based on Akaike and Schwarz information criteria values. The adequacy of FAR(1), ARIMA(1,1,2) and $\text{SARIMA}(2,1,1)(2,1,1)_{12}$ models were determined based on Autocorrelation and partial autocorrelation function residual plots. The out-sample FAR(1) model forecast captured and exhibited the seasonality and periodicity present in the Nigerian yearly inflation rate series which are not attained in the other models. Based on the forecast evaluation metrics obtained for the models, FAR(1) is the better model since its forecast evaluation metrics are lower. Conclusively, FAR(1) is the better model for forecasting the Nigerian inflation rate when the variation exhibited by series is considered.

Keywords: Fourier autoregressive model, Modelling, Forecasting, Inflation rate, variation

1. Introduction

Presently, inflation can be seen as the most dreadful economic variable affecting humanity globally Ibrahim *et al* 2022). It is wreaking havoc, surging economic activity, supply-chain disruptions, and soaring commodity prices (Nse *et al.* 2018; Katsaliaki *et al.* 2022). The sporadic rise of prices of commodities and services has become a threat to normal human livelihood and this has serious consequences on the purchasing power of citizens in many countries worldwide (World Economic Forum, 2021). Based on the complexity of inflation as an important macroeconomic variable which has caused global economic activity to experience a broad-based and sharper-than-expected slowdown, with inflation higher than seen in several decades (Wiri and lgbudu, 2022). The cost of living crisis and tightening financial conditions in most regions weigh heavily on the outlook. The global growth forecast is expected to move from 6.0 per cent in 2021 to 3.2 per cent in 2022 and 2.7 per cent in 2023 (Al Marhubi, 2021). This is the weakest growth profile since 2001 except for the global financial crisis and the acute phase of the COVID-19 pandemic. Therefore, global inflation is expected to rise from 4.7 per cent in 2021 to 8.8 per cent in 2022 but may rise to 6.5 per cent in 2023 (Agarwal and Kimball, 2023; International Monetary Fund, 2023). With this in-depth information, the developing countries which Nigeria is among and worst hit by the exponential rise of inflation need to critically monitor and put in place measures

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to combat the effects (Otu *et al.* 2014). To do this, future values of inflation must be obtained using an appropriate model. In recent years, many time series and economic time series models have by used to model and forecast inflation rate. Among these are ARIMA, Seasonal-ARIMA, autoregressive distributed lag (ARDL), time series regression, generalized (GARCH) and many more. These include the works of Feridun and Adebiyi, 2005; Bokhari and Feridun, 2006; Doguwa and Alade, 2013; Kelukume and Salami, 2014; Ikoku and Okany, 2017; Osuolale *et al.* 2017; Adubisi *et al.* 2018; Nyoni and Nathaniel, 2018; Olalude *et al.* 2021; Mustapha *et al.* 2021; Okorie *et al.* 2021; Nadum *et al.* 2022; Owan *et al.* 2022; Adenomon and Madu, 2022; Emwinloghosa *et al.* 2023; Awoyemi *et al.* 2024). Despite the good results obtained by several researchers over the years, a critical look at the variation exhibited by inflation rate time series datasets indicated that a model that can handle frequent seasonal and periodical changes will be more appropriate. In essence, this study will be used to carry out a Fourier autoregressive time series model building for the Nigerian inflation rate since this model could decompose time series data that exhibited seasonal and periodical variations and also take care of the interdependence between the series and its lags. Following the works of (Taiwo *et al.* 2019; Taiwo *et al.* 2020), the Fourier Autoregressive (FAR) model has four steps, these are identification, estimation, diagnostic and forecasting stages. The FAR model efficiency will contrasted with ARIMA and SARIMA models utilising the out-of-sample projection and forecast assessment metrics.

2. Materials and Methods

2.1 Fourier Autoregressive (FAR) model

A FAR approach is determined by assuming that $x_{k\omega+v} = \{x_{k\omega+v} \epsilon \mathbb{Z}\}$ is a cyclic stationary random procedure, then

$$x_{k\omega+v} = \varphi_0 + \sum_{i=1}^{p(v)} \left[\varphi_i(v) \cos 2\pi k/_{\omega} + \varphi_i^*(v) \sin 2\pi k/_{\omega}\right] x_{k\omega+v-} + \mu_{k\omega+v} \qquad (1)$$

wherein $\varphi_i(v)$ is a cyclic autoregressive parameter, ω is a count of periods, $\mu_{k\omega+v}$ is the residual having mean zero (0) and cyclic variance $\sigma_\varepsilon^2(v)$, v is a cyclic measure and k is the season measure. Since autocovariance function and mean are cyclic patterns of time having cyclic ω, the processes' initial and subsequent order moments become

$$E[x_t] = \mu(t) = \mu[t + k\omega] \text{ and } cov(x_t x_s) = \gamma(t, s) = \gamma(t + k\omega,\ s + k\omega) \qquad (2)$$

2.2 Fourier Autoregressive Model Building

There are four basic steps in time series analysis model building and these are

2.2.1 Identification of FAR model

Cyclic Autocorrelation *(CACF)* and Partial Autocorrelation function *(CPACF)* shall be utilised to identify the FAR approach which is to be fitted.

2.2.1.1 Cyclic-ACF

The CACF for a cyclic stationary series $\{\mu_{k\omega+v}\}$ slated in equation (1), where the residual is presumed independent can be expressed as

$$\gamma_{k\omega+v}(l) = cov(\mu_{k\omega+} \; y_{k\omega+v-}) = E[(\mu_{k\omega+v} - \mu_v)(\mu_{k\omega+v-i} - \mu_{v-l})] \qquad (3)$$

For season v in retrograde lag, $l \geq 0$, $CACF$ for time v at a retrograde lag $l \geq 0$ is perceived by

$$\rho_i(v) = \gamma_l(v) \Big/ \sqrt{\gamma_0(v)\gamma_0(v-l)}, \qquad l \geq 0 \qquad (4)$$

where $\gamma_0(v)$ is the variance for the v^{th}season.

2.2.1.2. Cyclic- PACF

The CPACF, $\emptyset_{ll}(v)$ can be pursued for measuring the exact relationship within x_{kw+v} and x_{kw+v-l} eliminating the impact of the previous observations and it is specified for whole numbers, $l \geq 1$ as

$$\emptyset_{ll} = corr\left[x_{k\omega+v}, x_{k\omega+v-l} \Big/ x_{kw+v-} \;, \dots, x_{kw+v-l+1}\right]$$

The CPACF is defined as

$$\varphi_{ll} = \frac{cov[(x_{k\omega+v} - \hat{x}_{k\omega+v}), (x_{k\omega+v-l} - \hat{x}_{k\omega+v-l})]}{\sqrt{var(x_{k\omega+v} - \hat{x}_{k\omega+v})}\sqrt{var(x_{k\omega+v-l} - \hat{x}_{k\omega+v-l})}} \qquad (5)$$

2.3 Estimation of FAR Model

The FAR coefficient shall be fitted utilising the Ordinary Least Square (OLS) procedure. Using the FAR(1) model

$$x_{t1} = \varphi_0 + \varphi_1 \frac{\cos}{12} x_{t1-1} + \varphi_1^* \frac{\sin 2}{12} x_{t1-1} + \mu_{k1-1} \qquad (6)$$

By making the error term the subject in (6), this gives

$$\mu_{k1-1} = y_{t1} - \varphi_0 - \varphi_1 \frac{\cos 2\pi}{12} x_{t1-1} - \varphi_1^* \frac{\sin 2\pi}{12} x_{t1-1} \qquad (7)$$

The residual sum of square of (7) is

$$\sum(\mu_{k1-1})^2 = \sum(x_{t1} - \varphi_0 - \varphi_1 \frac{\cos 2\pi}{12} x_{t1-1} - \varphi_1^* \frac{\sin 2\pi}{12} x_{t1-1})^2 \qquad (8)$$

By differentiating equation (8) with respect to φ_0, φ_1, $\varphi_1^*$and set to zero gives

$$\Sigma x_{t1} - n\varphi_0 - \varphi_1 \Sigma\left(\frac{\cos 2\pi}{12} x_{t1-1}\right) - \varphi_1^* \Sigma\left(\frac{\sin 2\pi}{12} x_{t1-1}\right) = 0 \qquad (9)$$

$$\Sigma(x_{t1}\frac{\cos 2\pi}{12} y_{t1-1}) - \varphi_0 \Sigma(\frac{\cos 2\pi}{12} x_{t1-1}) - \varphi_1 \Sigma\left(\frac{\cos 2\pi}{12} x_{t1-1}\right)^2 - \varphi_1^* \Sigma\left(\frac{\sin 2\pi}{12} x_{t1-1} \frac{\cos 2\pi}{12} y_{t1-1}\right) = 0 \qquad (10)$$

$$\Sigma\left(x_{t1}\frac{\sin 2\pi}{12} x_{t1-1}\right) - \varphi_0 \Sigma\left(\frac{\sin 2\pi}{12} x_{t1-1}\right) - \varphi_1 \Sigma\left(\frac{\sin 2\pi}{12} x_{t1-1} \frac{\cos 2\pi}{12} x_{t1-1}\right) - \varphi_1^* \left(\frac{\sin 2\pi}{12} x_{t1-1}\right)^2 = 0 \qquad (11)$$

By written equations (9) to (11) in matrix form gives

$$\begin{pmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_1^* \end{pmatrix} = \begin{pmatrix} n & \Sigma\left(\frac{\cos 2\pi}{12} x_{t1-1}\right) & \Sigma\left(\frac{\sin 2}{12} x_{t1-1}\right) \\ \varphi_0 \Sigma(\frac{\cos 2\pi}{12} x_{t1-1}) & \varphi_1 \Sigma\left(\frac{\cos 2\pi k}{12} x_{t1-1}\right)^2 & \varphi_1^* \Sigma\left(\frac{\sin 2\pi k}{12} x_{t1-1} \frac{\cos 2\pi}{12} x_{t1-1}\right) \\ \varphi_0 \Sigma(\frac{\sin 2\pi}{12} x_{t1-1}) & \varphi_1 \Sigma\left(\frac{\sin 2\pi}{12} x_{t1-1} \frac{\cos 2\pi}{12} x_{t1-1}\right) & \varphi_1^* \Sigma\left(\frac{\sin 2\pi}{12} x_{t1-1}\right)^2 \end{pmatrix}^{-1} \begin{pmatrix} \Sigma x_{t1} \\ \Sigma(x_{t1} \frac{\cos 2\pi}{12} x_{t1-1}) \\ \Sigma\left(x_{t1} \frac{\sin 2\pi}{12} x_{t1-1}\right) \end{pmatrix} \quad (12)$$

The estimated coefficients for the FAR(1) model are φ_0, φ_1, and φ_1^* respectively.

The cyclic Bayesian and Akaike information criteria modified based on the work of Akaike (1974) will be utilised for figuring out the optimal model to be chosen after estimation. These are expressed with

$$\text{AIC (C)} = n \, In \, \hat{\sigma}_\varepsilon^2(v) + 2C(v) \quad (13)$$

$$BIC(C) = In \, \hat{\sigma}_\varepsilon^2(v) + \frac{lnN}{N} C(v) \quad (14)$$

in which $C(v)$is the number of cyclic autoregressive parameters in the season and $\hat{\sigma}_\varepsilon^2(v)$ is a cyclic estimate of $\sigma_\varepsilon^2(v)$.

2.4 FAR Model Diagnostic Checking

Upon computation of coefficients, the model's suitability shall be evaluated by assessing if the model's presumptions are met. The fundamental presumption thus is $\{\varepsilon_t\}$ is noisy. Thus, by creating error terms ACF and PACF plots, a thorough examination regarding the fitted error terms shall be conducted to determine if the error terms were white noise**.**

2.5 Forecasting with FAR Model

Given a FAR(1) model as

$$x_{t1} = \delta + \varphi_1 cosz(x_{t1-1}) - \varphi_1^* sinz(x_{t1-1}) + \mu_t$$

$$(1 - \varphi_1 cosc - \emptyset_1^* sinc)(x_{t1} - \delta) = \mu_t \quad (15)$$

where $c = \frac{2\pi k}{\omega}$ and δ is a fixed variable.

Equation (15) could be expressed as

$$x_{t1} - \delta = [(\emptyset_1 coscx_{t1-1} - \emptyset_1^* sincx_{t1-1}) - \delta)] \quad (16)$$

The forecast formula is provided in a broad context as

$$\hat{x}_{t1}(l) = \delta + [(\emptyset_1 coscx_{t1}(l-1) - \delta) + (\emptyset_1^* sincx_{t1}(l-1) - \delta)]$$

$$= \delta + [\left(\emptyset_1^l coscx_{t1}(l-1) - \delta\right) + (\emptyset_1^{*l} sincx_{t1}(l-1) - \delta\big) \quad l \geq 1 \quad (17)$$

2.5.1 FAR Model Forecast Evaluation

After the forecast is produced, it is evaluated to see if the variable forecast's actual outcomes match the observations. Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE) are the forecast evaluation metrics that are utilised.

These are expressed with

$$RMSE = \sqrt{\frac{1}{tv+1}\sum_{tv=1}^{p-1}(\hat{x}_{tv} - x_{tv})^2} \quad (18)$$

$$MAPE = \sum_{tv=1}^{p-1}\left|\frac{\hat{x}_{tv}-x_{tv}}{\hat{x}_{tv}}\right| \quad (19)$$

$$MAE = \frac{1}{tv+1}\sum_{tv=1}^{p-1}(\hat{x}_{tv} - x_{tv})^2 \quad (20)$$

where $tv = 1, 2, \dots, p-1$.The framework performs more effectively in forecasting when RMSE, MAPE, and MAE values are much smaller (Olatayo *et al.* 2015).

2.6 Augmented Dickey-Fuller Unit Root Test

The Augmented Dickey-Fuller (ADF) procedure will be utilised for attaining stationarity for the series under consideration. This involved the use of associated standard errors as well that will be compared with the test statistic using the appropriate values in the Dickey-Fuller table.

2.7 Autoregressive Integrated Moving Average (ARIMA) Model

This is a univariate time series model that consists of an autoregressive polynomial, an order of integration (d) and a moving average polynomial. The usual forms of $AR(p)$ and $MA(q)$ are written as

$$x_t = \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \cdots + \emptyset_p x_{t-p} + e_t \quad (21)$$

and

$$x_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q} \quad (22)$$

where $\emptyset$ and θ are the autoregressive and moving average parameters respectively. x_t is the observed value at time t and ε_t is the value of the random shock at time t. It is assumed to be independently and identically distributed with a mean of zero and a constant variance (σ^2). ARMA $(p.q)$ model comprised of AR and MA models, in which the current value of the time series is defined linearly in terms of its previous values as well as current and previous error series.

The ARMA (p, q) model is given in equation (23) as

$$x_t = \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \cdots + \emptyset_p x_{t-p} + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q} \quad (23)$$

Equation (23) can be simplified by a backward shift operator B to obtain

$$(B)\nabla^d x_t = \theta(B)w_t \quad (24)$$

Equation (24) can therefore be expressed as $ARMA(p, d, q)$ where $\nabla^d = (1-B)^d$ with $\nabla^d y_t$ and d^{th} consecutive differencing.

2.8 Seasonal-ARIMA model

An expansion of the ARIMA approach created by Box and Jenkins (1970) is the seasonal ARIMA put forth by Box *et al.,* (2013). This model is used to reflect and obtain the features of seasonal variation in a given time series.

Generally, the time series $\{x_t\}$ utilizes a lag operator B to process $SARIMA(p,d,q)(P.D.Q)s$.

A seasonal ARIMA model may be written as:

$$\emptyset_p(B)\Phi_P(B^S)^d(1-B^S)x_t = \theta_q(B)\Theta_q(B^S)\varepsilon_t \qquad (25)$$

In equation (25), B is a lag operator defined as $B^k x_t = x_{t-k}$;

$$\emptyset_p(B) = 1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_P B^P \qquad (26)$$

$$\Phi_p(B^S) = 1 - \Phi_s B^s - \Phi_2 B^{2s} - \cdots - \Phi_P B^{Ps}$$

$$\Theta_q(B) = 1 - \Theta_1 B - \Theta_2 B^2 - \cdots - \Theta_q B^P$$

$$\Theta_q(B^S) = 1 - \Theta_s B^s - \Theta_2 B^{2s} - \cdots - \Theta_q B^{Qs} \qquad (27)$$

where $\phi(B)$ and $\theta(B)$ are polynomials of order p and q respectively; $\Phi_p(B^S)$ and $\Theta_q(B)$ are polynomials in B of degrees P and Q , respectively; p is the order of non-seasonal autoregression; d is the number of regular differences; q is the order of non-seasonal moving average; P is the order of seasonal autoregression; D is the number of seasonal differences; Q is the order of seasonal moving average; and S is the length of season.

3. Results and Discussion

3.1 Data Exploration

The secondary data used was obtained from the National Bureau of Statistics (NBS) E-library (2022) from 1960 to 2022. The descriptive statistics results of the Nigerian yearly inflation rate are given in Table 1 and this indicated 15.85 as the mean with 15.03 as the standard deviation. The inflation rate lowest occurrence was in 1967 and the maximum was 72.84 in 1995 respectively.

The time plot of the Nigerian yearly inflation series is displayed in Figure 1 and this shows that the inflation series is non-stationary but exhibits seasonal and cyclical variations as well. Therefore, this indicated that the nature of the pattern of series informs the use of the Fourier Autoregressive (FAR) model that can handle the variations simultaneously. The augmented Dickey-Fuller test was utilised to attain stationarity. Table 2 was utilised to signify that the series is not stationary at its level but stationary at the first difference that is I(1) at 1, 5 and 10 percent levels of significance respectively.

3.2 Fourier Autoregressive (FAR) model identification and estimation

An examination of the $CACF$ and $CPACF$ for inflation rate series signified stability of $CACF$ and a cut-off at lag 2 for $CPACF$. Based on this result in Figure 2, the chosen tentative models for modelling and forecasting Nigerian yearly inflation rate series are FAR(1), FAR(2) and FAR(3). The parameters of the FAR(1), FAR(2), and FAR(3) models were obtained using the ordinary least square procedure. In light of the lowest possible values of the Cyclic Akaike and

Bayesian Information criteria provided in Table 3, the FAR(1) model was determined to be the most suitable one.

Table 1. Descriptive Statistics of Nigerian yearly Inflation rate

Variable	Mean	SE Mean	St Dev.	Minimum	Q1	Median	Q3	Maximum
Inflation rate	15.85	1.89	15.03	-3.73	7.44	12.10	17.82	72.84

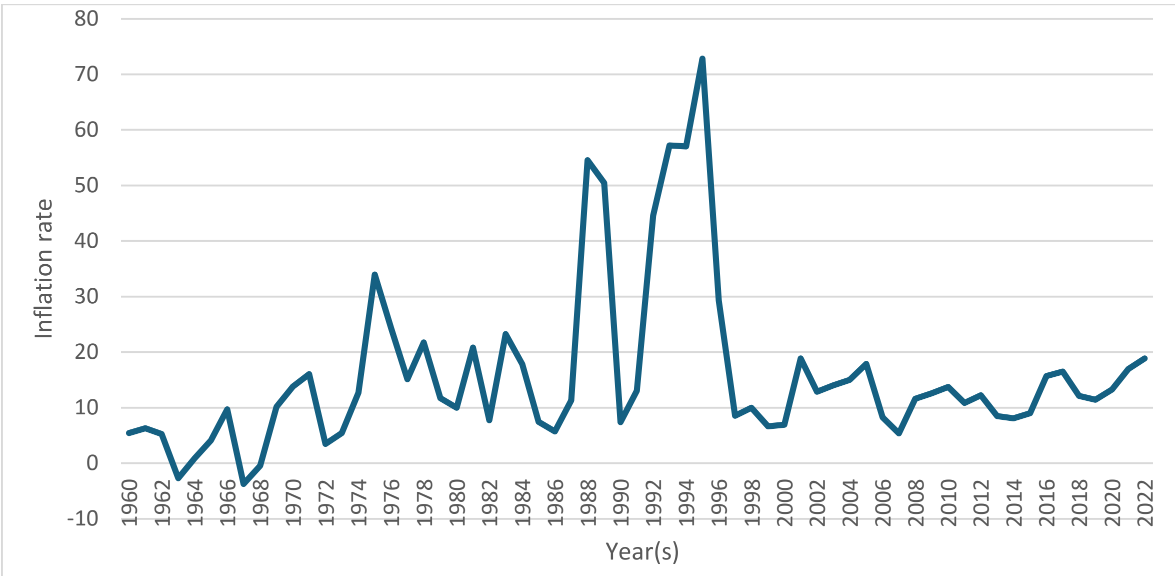

Figure 1. Time plot of Nigerian yearly inflation rate from 1960 to October 2022

Table 2. Stationarity test results

		t-Statistic	Prob.*
ADF		-6.206344	0.0000
Test critical values:	1% level	-3.548208	
	5% level	-2.912631	
	10% level	-2.594027	

CACF	CPACF		AC	PAC	Q-Stat	Prob
. |***** |	. |***** |	1	0.635	0.635	26.660	0.000
. |** |	**| . |	2	0.261	-0.240	31.220	0.000
. |*. |	. |** |	3	0.177	0.221	33.357	0.000
. |*. |	. | . |	4	0.167	-0.026	35.298	0.000
. |*. |	. |*. |	5	0.211	0.184	38.446	0.000
. |** |	. | . |	6	0.234	0.013	42.377	0.000
. |*. |	.*| . |	7	0.129	-0.090	43.585	0.000
. | . |	.*| . |	8	-0.034	-0.128	43.670	0.000
.*| . |	. | . |	9	-0.084	0.008	44.209	0.000
.*| . |	. | . |	10	-0.067	-0.055	44.557	0.000
.*| . |	. | . |	11	-0.075	-0.056	45.000	0.000
. | . |	. |*. |	12	-0.028	0.091	45.062	0.000
. | . |	. | . |	13	-0.006	-0.023	45.065	0.000
. | . |	. | . |	14	-0.047	0.012	45.251	0.000
.*| . |	.*| . |	15	-0.125	-0.127	46.588	0.000
.*| . |	. |*. |	16	-0.090	0.128	47.300	0.000
. | . |	. | . |	17	0.031	0.069	47.388	0.000
. | . |	.*| . |	18	0.005	-0.154	47.390	0.000
. | . |	. | . |	19	-0.051	0.016	47.634	0.000
.*| . |	.*| . |	20	-0.107	-0.122	48.724	0.000

Figure 2. Cyclic Autocorrelation and Partial autocorrelation for Inflation series

Table 3. Information Criteria for Inflation Rate Series

Information Criteria	FAR(1)	FAR(2)	FAR(3)
PAIC	8.024567 *	8.298955	8.157892
PBIC	8. 202095 *	8.401009	8.387152

The fitted FAR|(1) model estimated using ordinary least square and chosen based on the information criteria is given in equation (28) as

$$y_{Inflation\,rate} = 15.75757 - 3.998603\cos\left(\frac{2\pi k}{\omega}\right)y_{t-1} - 0.417016\sin\left(\frac{2\pi k}{\omega}\right)y_{t-1} \quad (28)$$

3.3. Fourier Autoregressive (FAR) model diagnostics test

A critical look at the residual $CACF$ and $CPACF$ for fitted FAR(1) in Figure 3 signified that the residuals do not have any usual structure and are significant at $\alpha = 0.05$. That is, all the points fall within the 5% significance limit. Therefore, the fitted FAR model is considered suitable when forecasting the Nigerian yearly inflation rate.

3.4 FAR model forecasting

The fitted FAR(1) is used to obtain an out-sample forecast for the Nigerian yearly inflation rate from 2023 to 2037. The out-sample forecast values and time plot are given in Table 4 and Figure 4 respectively. This signified Nigerian inflation rate has continuous periodic variation and is a near replica of the original series from 1960 to 2037. In essence, the fitted Fourier autoregressive model out-sample forecast signified the model is suitable for handling complex, nonlinear dynamics, structural breaks, and uncertainty usually present in Inflation rate time series data.

<table>
<tr><th>CACF</th><th>CPACF</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>. |**** |</td><td>. |**** |</td><td>1</td><td>0.569</td><td>0.569</td><td>21.372</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>**| . |</td><td>2</td><td>0.184</td><td>-0.206</td><td>23.648</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>3</td><td>0.103</td><td>0.145</td><td>24.369</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>4</td><td>0.129</td><td>0.045</td><td>25.516</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>. | . |</td><td>5</td><td>0.143</td><td>0.056</td><td>26.958</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>. |*. |</td><td>6</td><td>0.180</td><td>0.113</td><td>29.291</td><td>0.000</td></tr>
<tr><td>. |*. |</td><td>.*| . |</td><td>7</td><td>0.096</td><td>-0.107</td><td>29.963</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>8</td><td>-0.038</td><td>-0.072</td><td>30.073</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>9</td><td>-0.052</td><td>0.024</td><td>30.276</td><td>0.000</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>10</td><td>-0.023</td><td>-0.041</td><td>30.318</td><td>0.001</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>11</td><td>-0.072</td><td>-0.094</td><td>30.728</td><td>0.001</td></tr>
<tr><td>. | . |</td><td>. |*. |</td><td>12</td><td>-0.040</td><td>0.081</td><td>30.859</td><td>0.002</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>13</td><td>0.008</td><td>0.001</td><td>30.864</td><td>0.004</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>14</td><td>-0.004</td><td>0.005</td><td>30.865</td><td>0.006</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>15</td><td>-0.128</td><td>-0.168</td><td>32.269</td><td>0.006</td></tr>
<tr><td>.*| . |</td><td>. |*. |</td><td>16</td><td>-0.075</td><td>0.145</td><td>32.755</td><td>0.008</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>17</td><td>0.049</td><td>0.069</td><td>32.966</td><td>0.011</td></tr>
<tr><td>. | . |</td><td>.*| . |</td><td>18</td><td>0.036</td><td>-0.082</td><td>33.086</td><td>0.016</td></tr>
<tr><td>. | . |</td><td>. | . |</td><td>19</td><td>-0.016</td><td>-0.009</td><td>33.111</td><td>0.023</td></tr>
<tr><td>.*| . |</td><td>.*| . |</td><td>20</td><td>-0.067</td><td>-0.071</td><td>33.535</td><td>0.029</td></tr>
</table>

Figure 3. Residual Cyclic Autocorrelation and Partial autocorrelation for Inflation series

Table 4. Forecast of Nigerian inflation rate series from 2023 to 2037

Year(s)	FAR	ARIMA	SARIMA
2023	11.7590	18.2835	-4.7921
2024	12.5532	18.2494	34.6072
2025	16.8538	18.3051	31.7011
2026	19.5362	18.3910	-9.6853
2027	14.8583	18.4843	-2.1705
2028	19.5541	18.5817	32.0356
2029	14.6394	18.6801	40.0990
2030	17.8514	18.7787	46.4414
2031	11.9889	18.8775	54.2959
2032	11.7483	18.9764	12.5356
2033	12.6367	19.0752	-1.1249
2034	17.8194	19.1740	5.06680
2035	12.1484	19.2729	-8.77490
2036	12.3839	19.3717	-16.7610
2037	14.7404	19.4705	-5.89370

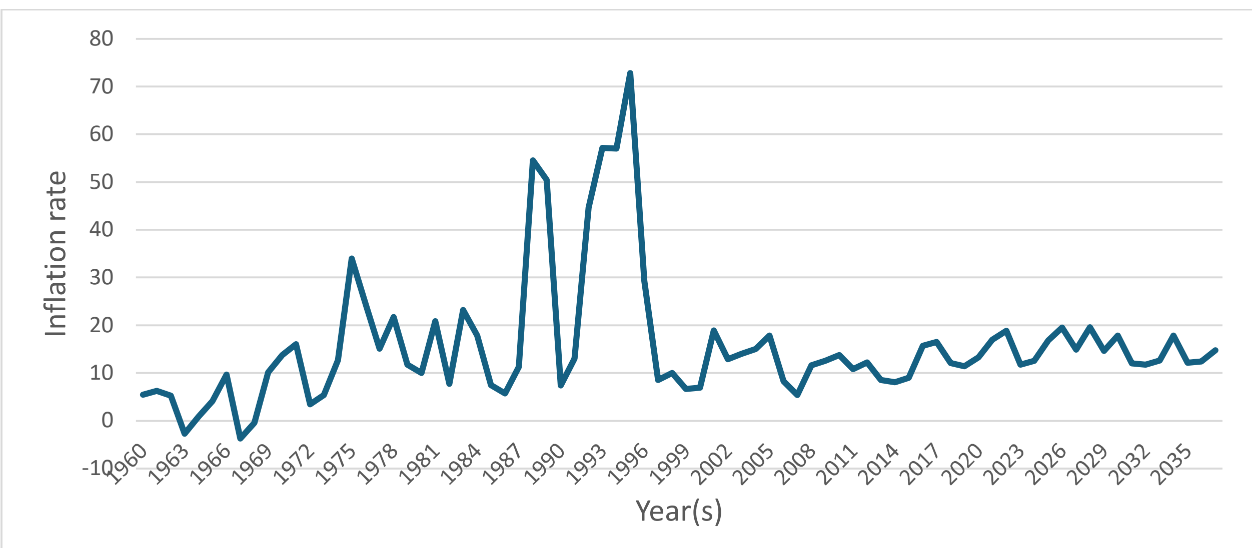

Figure 4. Time plot of Nigerian inflation rate forecast from 1960 to 2037 (FAR model)

3.4. Comparison of FAR, ARIMA and SARIMA models

The Nigerian yearly inflation rate is stationary at $d = 1$ utilising the Augmented Dickey-Fuller test results in Table 2. Thereafter, ARIMA and SARIMA models were identified using ACF and PACF. The ACF plot tailed off at lag 1 and PACF cut-off after lag 2, hence p = 1, 2 and q = 1 or p = 2 and q = 1, 2 were picked.

The tentative models for ARIMA are

$$ARIMA(1,1,1), ARIMA(1,1,2), ARIMA(2,1,1) \; and \; ARIMA(2,1,2)$$

For the SARIMA model,

$$SARIMA(1,1,2) \times (1,1,1)_{12}, SARIMA(2,1,2) \times (1,1,2)_{12}, SARIMA(2,1,1) \times (2,1,1)_{12} \; and \; SARIMA(1,1,2) \times (1,1,2)_{12}$$

were the chosen tentative models.

The ordinary least square procedure was used to fit the four proposed approaches for ARIMA and SARIMA models. Utilising the lowest values of the fitted Schwarz and Akaike information criteria, better frameworks for the annual inflation rate were selected. The optimal models for the Nigerian yearly inflation rate are $ARIMA(1,1,2)$ and $SARIMA(2,1,1) \times (2,1,1)_{12}$ models. The residuals of these approaches do not have any usual structure and are significant at $\alpha = 0.05$. That is, all the points fall within the 5% significance limit based on ACF and PACF residual plots.

To ascertain the reasons while Fourier Autoregressive technique is a better model for the Nigerian inflation rate forecast, ARIMA(1,1,2) and SARIMA$(2,1,1)(2,1,1)_{12}$ models forecast values given in Table 4 and Figure 4 were compared with that of FAR(1) model. Based on the comparison, ARIMA(1,1,2) and SARIMA$(2,1,1)(2,1,1)_{12}$ models are not appropriate for forecasting the Nigerian inflation rate since the forecasted values from these models could not capture and reflect the cyclical and periodicity that is present in the inflation rate series. While SARIMA$(2,1,1)(2,1,1)_{12}$ model captured and exhibited the seasonality in the Nigerian inflation rate series but the periodicity in the series were not resolved. However, the FAR(1) model forecast in Table 4 and Figure 4 captured and exhibited the seasonality and periodicity present in the inflation rate series. As well, the FAR(1) model showed a continuous periodic movement and close reflection to the original series from 2023 to 2037. The values of the forecast evaluation for the FAR(1) model in Table 5 showed as well the consistency of the forecast since the values were relatively low. Hence, the Fourier Autoregressive model is adequate and suitable for modelling and forecasting Nigerian rainfall series.

The forecast evaluation metrics obtained from FAR(1), ARIMA(1,1,2) and SARIMA$(2,1,1)(2,1,1)_{12}$ models are displayed in Table 5. The MAE, RMSE and MAPE values in Table 5 signified that the FAR(1) model is the better model since its forecast evaluation metrics are lower than that of the ARIMA(1,1,2) and SARIMA$(2,1,1)(2,1,1)_{12}$ models. Therefore, FAR(1) is the better model for forecasting the Nigerian inflation rate from 1960 to 2037.

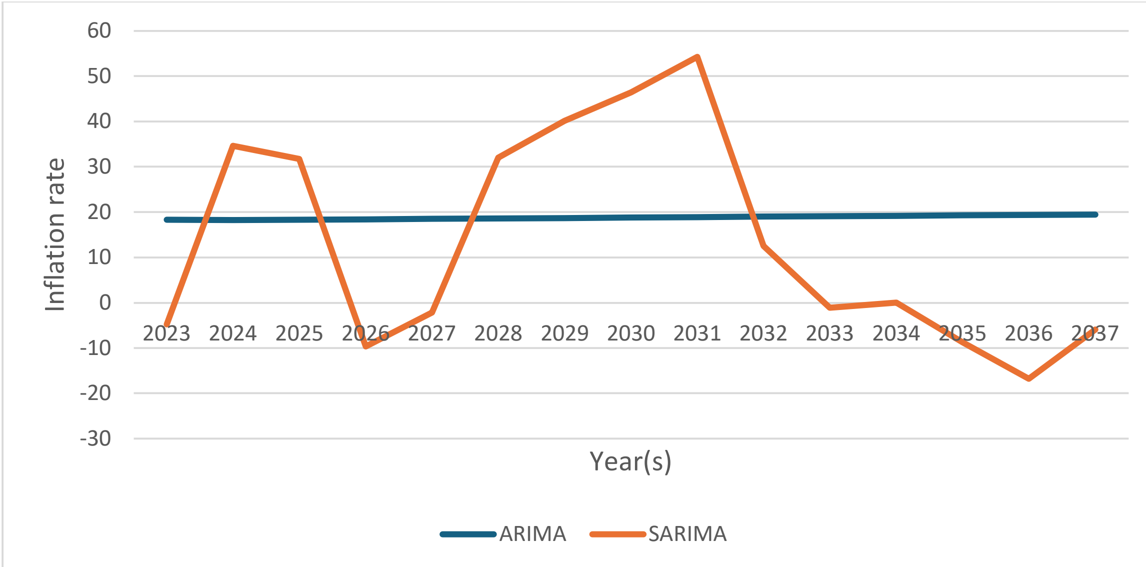

Figure 4. Time plot of Nigerian inflation forecast from 2023 to 2037 (ARIMA and SARIMA models)

Table 5. Forecast Evaluation metrics for FAR, ARIMA and SARIMA models

Forecast Evaluation Metrics	$FAR(1)$	$ARIMA(2,1,5)$	$SARIMA(2,1,5) \times (1,1,1)_{12}$
PMAE	4.627743		
PRMSE	5.037131		
PMAPE	22.49446		
MAE		10.480962	6.085470
RMSE		9.882820	7.497472
MAPE		32.69064	30.24077

4. Conclusion

Several researches have been carried out to analyse and forecast inflation rate in Nigeria and globally. However, most researchers do not take into cognisance the presence of cyclical and periodic variation in inflation rate time series data. Therefore, FAR, ARIMA and SARIMA models were used to analyse and forecast the Nigerian inflation rate to consider stationarity, seasonality and periodicity in the series. From the results obtained, the FAR(1) model was identified to be the better model since its forecast follows the pattern of previous years and also indicates the situation of inflation rate pattern in Nigeria. The forecast evaluation metrics obtained from FAR(1) are lower than those of ARIMA(1,1,2) and SARIMA$(2,1,1)(2,1,1)_{12}$ models. Therefore, FAR(1) is the better model for forecasting the Nigerian inflation rate when the variation exhibited by series is considered. The study is used to recommend that measures and policies should be put in place to cushion the negative effects of inflation on the economy, there is a need to have an insight into the future value of inflation to design appropriate policy measures to cushion the effects of inflation on the economy. Future studies on macroeconomic variables should consider the possibility of proposing more models that further improve the forecasting of the inflation rate.

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