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Meenakshi's-Gompertz Distribution and Its Properties

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ABSTRACT
The discussion revolves around the broader concept of exponential family distributions, exploring their applications in survival analysis and reliability engineering. Various distributions within this family, such as the Weibull, Rayleigh, and Gompertz distributions, are examined in terms of their suitability for modelling different phenomena, including instantaneous failure events, independent sums of events, and decreasing processes over time. Additionally, a newly derived distribution, termed the Meenakshi's-Gompertz distribution, is introduced, with its parameters interchangeably representing scale and shape properties.

Keywords: *Distribution, mean, variance, moment generating function and MLE.*

1. Introduction

When we studied the concept of uncertainty, we introduced probability density and its distribution function of random variables. In the parametric family of density functions is a collection of density functions that is indexed by quantity called a parameter (Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). The parameters are introduced in most of the families of distributions that are mean, variance, and moments, and also introduced moment generation function, characteristics function, cumulant generating function, entropy, survival, and hazard function of distribution. The number of potential distribution models is very large; in practice, a relatively small number have come to prominence, either because they have desirable mathematical characteristics or because they relate particularly well to some slice of reality or both. Evans, M., Hastings, N., Peacock, B., & Forbes, C. (2011).
Example: the family of exponential distribution. If random variable is time between the two succeeding events. It is concentrated on occurrence and also follows the memory less nature, such distribution is simply called as exponential. It is applicable for various flieds of reliability engineering, queuing theory, survival analysis, Telecommunications, Economics and finance, renewal processes and statistical inference.

The generalization of exponential distribution extended to one shape parameter α that is called gamma distribution. if random variable follows the interval time between the two events which is increasing, decreasing and not constant rate. It is applicable for the field of queuing theory reliability engineering, health and medical science, economics and finance, traffic engineering, insurance and actuarial science image processing, environmental studies. It reduces to erlang where scale parameter at proportion (Eric U., Oti Michael O. Olusola, and Francis C. Eze2021).

A special case of versatile probabilities distribution is Weibull distribution. it is also generalization of gamma distribution, were shape parameter as power function of scale parameter. It is used for various field of reliability engineering, survival analysis, wind energy, material science, medical science, quality control, economics and finance. It is suitable for

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model designing to the environmental studies of extreme event such as flood, droughts or other natural disasters. This distribution is not a universal fit for all situation. When scale parameter α=2 (Hadeel S. Klakattawi 2021)

Another one parameter case of exponential family of distribution such as a random variable as sum of independent events peculiar situation of wind speed, power received by radar from the scatter surface. The distribution of the random variable is called as Rayleigh. It is the particular case of Weibull distribution when shape parameter α=2. It is application for the received signal is sum of many independent randomly phased reflections, leading to a Rayleigh distribution amplitude. Wide application in real world, that are wireless communication, radar systems wind speed modelling, seismology, image processing physics and engineering wireless sensor networks.

Another one of the particular cases of exponential distribution in which variable expressed on the curve liners form which is introduced by Benjamin Gompertz in 1825. The model for lifetime for systems with increasing failure rate over time. It is applicable for modelling of mortality rate in population, survival analysis, actual science, biological and medical sciences and reliability engineering.

Reliability engineering of increasing failure rate. In the Survival analysis, it is suitable for scenarios where the hazard rate (instantaneous failure rate) increases exponentially with time. In the Actuarial science it is used for modelling mortality rate and analysing the life expectancy. In the Biological and medical science, it is used model the growth of tumours or the distribution of lifetimes for certain organisms. It has been used in studies related to aging and mortality. The Gompertz distribution is established that the specifics characteristics of the data or phenomenon being modelled.

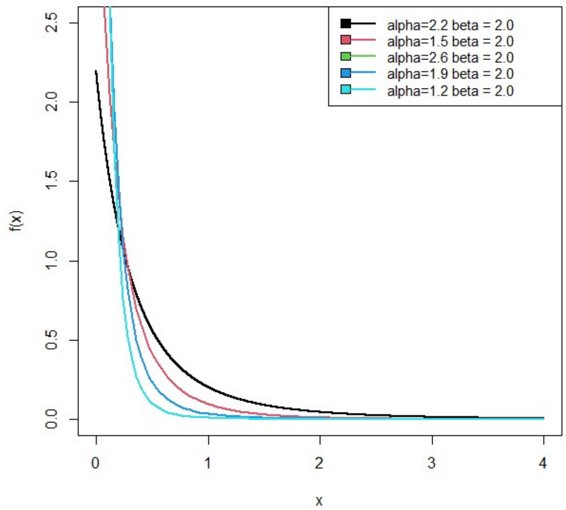
2. Meenakshi's Gompertz distribution

The motivation of the newly derived distribution, which is the situation of complications In the biological study, the viral replication processes are rapid change at the instantaneous time period, where the count level of the infected patient suddenly decreases. The newly derived distribution is suitable for this kind of study. This distribution is exclusively characterized by the rapid growth of viral replication. In particular, the exponential distribution follows the concept of rate of interval between the events. The generalization of exponentials, such as the gamma distribution, which is not a constant rate but either increasing or decreasing in nature. In that situation, the specific case of the exponential family of the distribution, namely the Weibull distribution, It is suitable for instantaneous failure of the event in the wide-range phenomenon. In the particular case of the Weibull distribution, such as the Rayleigh distribution, the event over period is a floating and independent sum of events. One of the exponential families of the distribution is the Gompertz distribution, the event-decreasing nature over time. It is suitable for the count-decreasing nature of the infected patient, but the newly derived distribution is suitable for rapid growth of the viral replication process when sudden decline of cells. When cell distribution follows, Gompertz (J. A. Adewara, J. S. Adeyeye, and C. P. 2019). The newly derived distribution. Interchanging the scale parameter as shape parameter and shape parameter as scale parameter, the new distribution is named Meenakshi's-Gompertz (MG) distribution (Nadarajah, S., & Kotz, S. (200).

3.1. The random variable is said to have Meenakshi's Gompertz distribution and it is given by

$$f(x,\alpha,\beta) = \alpha\beta e^{\left(\alpha e^{-\beta x} - \beta x - \ \right)} \qquad \alpha, \beta > 0 \qquad \ldots.(1)$$

where α is shape parameter and β is scale parameter



Figures.1 pdf plot of MG distribution

3.2. The cumulative distribution function is given by

$$F(x) = \int_0^x \alpha\beta e^{(\alpha e^{-\beta x} - \beta x - \alpha)}\, dx$$

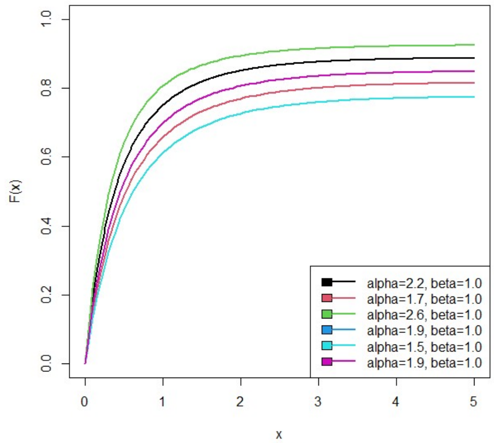
$$= \alpha\beta e^{-\alpha} \int_0^x e^{(\alpha e^{-\beta x})} * e^{-\beta x}\, dx$$

Let

$e^{-\beta x} = z$

$-\beta e^{-\beta x} dx = zdz$

$F(x) = \left[1 - e^{(\alpha[e^{-\beta x} - 1])}\right]$ ....(2)



Figures.2 cdf plot of MB distribution

3.3. The mean of the Meenakshi's-Gompertz distribution is given by

$$E[X] = \int_0^\infty x.\alpha\beta e^{(\alpha e^{-\beta x}-\beta x-\alpha)}\,dx$$

$$E[X] = \sum_{i=0}^{\infty} \frac{\alpha\beta e^{-\alpha}\alpha^i}{i!}\left[\frac{gamma(2)}{\left(\beta(i+1)\right)^2}\right]$$

Where

$$e^{\alpha e^{-\beta x}} = \sum_{i=0}^{\infty} \frac{\left[\alpha e^{-\beta x}\right]^i}{i!}$$

Table :1 Mean with various values of β

β	$E[x]$
1	1.383257
2	0.691629
3	0.461086
4	0.345814
5	0.276652
6	0.230543
7	0.197608
8	0.172907
9	0.153695
10	0.153695

Table :2 Mean with various values of α

α	$E[x]$
0.5	1.383257
0.6	0.691629
0.7	0.461086
0.8	0.345814
0.9	0.276652
0.1	0.230543
0.11	0.197608
0.12	0.172907
0.13	0.153695
0.14	0.138326

3.4. The second raw moment is given

$$E[X^2] = \int_0^\infty x^2.\alpha\beta e^{(\alpha e^{-\beta x}-\beta x-\alpha)}\,dx$$

$$= \alpha\beta e^{-\alpha}\int_0^\infty x^2.e^{(\alpha e^{-\beta x}-\beta x)}\,dx$$

$$E[X^2] = \sum_{i=0}^{\infty}\frac{\alpha\beta e^{-\alpha}\alpha^i}{i!} * \left[\frac{\Upsilon 3}{\left(\beta(i+1)\right)^3}\right]$$

Where

$$e^{\alpha e^{-\beta x}} = \sum_{i=0}^{\infty}\frac{\left[\alpha e^{-\beta x}\right]^i}{i!}$$

3.5. Variance for Meenakshi's-Gompertz distribution is given by

$$V(X) = \sum_{i=0}^{\infty}\frac{\alpha\beta e^{-\alpha}\alpha^i}{i!} * \left[\frac{\Upsilon 3}{\left(\beta(i+1)\right)^3}\right] - \left\{\sum_{i=0}^{\infty}\frac{\alpha\beta e^{-\alpha}\alpha^i}{i!}\left[\frac{\Upsilon 2}{\left(\beta(i+1)\right)^2}\right]\right\}^2$$

Table: 3 Variance for Meenakshi's-Gompertz distribution with respective α

α	$v(x)$
0.6	0.293818
0.7	0.626000
0.8	0.773071
0.9	0.859352
0.1	0.916508
0.11	0.956137
0.12	0.983655
0.13	1.002089
0.14	1.013377

Table: 4 Variance for Meenakshi's-Gompertz distribution with respective β

β	$v(x)$
1	1.250866e+05
2	1.146197e+11
3	1.339073e+17
4	1.696866e+23
5	2.227939e+29
6	2.976803e+35
7	4.012659e+41
8	5.430782e+47
9	7.358091e+53
10	9.961528e+59

3.6. The moment generating function

The moment generating function is given by

$$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx}.\,\alpha\beta e^{(\alpha e^{-\beta x}-\beta x-\alpha)}\,dx$$

$$= \sum_{i=0}^{\infty} \frac{\left[\alpha^{i+1}\beta e^{-\alpha}\right]}{i!}\left[\frac{e^{-x(\beta+\beta i-\ )}}{-(\beta+\beta i-t)}\right]_0^\infty$$

$$M_x(t) = \sum_{i=0}^{\infty} \frac{\left[\alpha^{i+1}\beta e^{-\alpha}\right]}{i!}\left[\frac{1}{(\beta+\beta i-t)}\right]$$

Where

$$e^{\alpha e^{-\beta x}} = \sum_{i=0}^{\infty} \frac{\left[\alpha e^{-\beta x}\right]^i}{i!}$$

3.7. The characteristics function

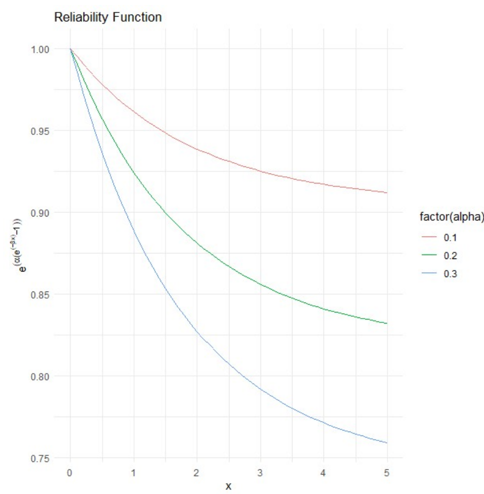
The characteristics function is given by

$$\varphi_x(t) = E\left(e^{jtx}\right) = \sum_{i=0}^{\infty} \frac{\left[\alpha^{i+1}\beta e^{-\alpha}\right]}{i!}\left[\frac{1}{(\beta + \beta i - jt)}\right]$$

3.8. The reliability functions

The reliability function is given by

$$R_x(t) = 1 - F(x)$$
$$= 1 - \left[1 - e^{(\alpha[e^{-\beta x} - 1])}\right]$$
$$= e^{(\alpha[e^{-\beta x} - 1])}$$



3.9. The Hazard functions

The Hazard function is given by

$$H_x(t) = \frac{f(x)}{R_x(t)}$$
$$= \frac{\alpha\beta e^{(\alpha e^{-\beta x} - \beta x - \alpha)}}{e^{(\alpha[e^{-\beta x} - 1])}}$$

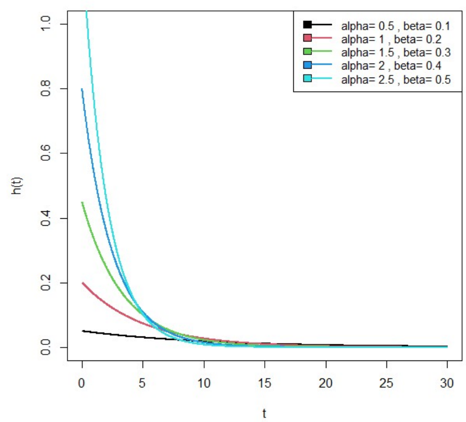


Fig.3: Hazard Rate Function Plot for MG Distribution

The likelihood function is given by

$$l(x,\alpha,\beta)=\prod_{i=1}^{n}\alpha\beta e^{(\alpha e^{-\beta x_i}-\beta x_i-\alpha)}$$
$$=(\alpha\beta)^n e^{n\alpha}e^{-\beta\sum_{i=1}^{n}x_i}e^{\left(\alpha e^{-\sum_{i=1}^{n}x_i}\right)}$$

Log-likelihood function is given by

$$\log[l(x,\alpha,\beta)]=\log\left[\prod_{i=1}^{n}\alpha\beta e^{(\alpha e^{-\beta x_i}-\beta x_i-\alpha)}\right]$$
$$=\log\left[(\alpha\beta)^n e^{n\alpha}e^{-\beta\sum_{i=1}^{n}x_i}e^{\left(\alpha e^{-\sum_{i=1}^{n}x_i}\right)}\right]$$

3.10 Maximum Likelihood Estimation

MLE is given by

$$\frac{\partial\log[l(x,\alpha,\beta)]}{\partial\alpha}=\frac{\partial\log\left[(\alpha\beta)^n e^{n\alpha}e^{-\beta\sum_{i=1}^{n}x_i}e^{\left(\alpha e^{-\sum_{i=1}^{n}x_i}\right)}\right]}{\partial\alpha}$$
$$\frac{\partial\log[l(x,\alpha,\beta)]}{\partial\beta}=\frac{\partial\log\left[(\alpha\beta)^n e^{n\alpha}e^{-\beta\sum_{i=1}^{n}x_i}e^{\left(\alpha e^{-\sum_{i=1}^{n}x_i}\right)}\right]}{\partial\beta}$$

3.11 Order statistics

$$f_X k(x)=\frac{n!}{(k-1)!\,(n-k)!}[F_X(x)]^{k-1}[1-F_X(n)]^{n-k}f_X(x)$$
$$=\frac{n!}{(k-1)!\,(n-k)!}\left[1-e^{(\alpha[e^{-\beta x}-1])}\right]^{k-1}\left[e^{(\alpha[e^{-\beta x}-1])}\right]^{n-k}\left[\alpha\beta e^{(\alpha e^{-\beta x}-\beta x-\alpha)}\right]$$

This expression combines the probabilities associated with the position of $X_K$considering the cumulative probabilities of preceding and succeeding order statistics. Understanding this formula provides valuable insights into the distributional properties of order statistics, contributing to various statistical applications such as hypothesis testing, reliability analysis, and survival studies.

4. The simulation studies

It is difficult to formulate MLE so it is calculated by using R software. The R code is provided below, a simulation is used to generate artificial or synthetic data that follows a particular mathematical model. This synthetic data is then used for the purpose of analysis, optimization, and parameter estimation. The algorithm of simulation process is given below:

1.Mathematical Model: (**model_function**) The model function takes parameters alpha and beta, where alpha represents the shape parameter, and beta represents the scale parameter.
The model function is defined as:

```
model_function <- function(x, alpha, beta) {
  alpha * beta * exp(-alpha * exp(-beta * x))
}
```

2. Simulation of Data: The simulated data is generated using the model function with known or "true" values of parameters (alpha_true and beta_true). Additionally, random noise is added to the data using **rnorm** to introduce variability.

```
set.seed(123)
x_sim <- seq(0, 10, length.out = sample_size)
alpha_true <- 2
beta_true <- 0.5
y_sim <- model_function(x_sim, alpha = alpha_true, beta = beta_true) + rnorm(sample_size,
mean = 0, sd = 0.2)
simulated_data <- data.frame(x = x_sim, y = y_sim)
```

3.**Analysis with Simulated Data:** The simulated data (**simulated_data**) is then used in the optimization process to estimate the parameters (alpha and beta) that would best fit the observed data. The optimization is performed using the **optim** function in R.

```
optim_result <- optim(par = initial_params, fn = function(p) -log_likelihood_model(p), method
= "Nelder-Mead")
```

The negative log-likelihood is minimized to find the parameter values (**alpha_hat** and **beta_hat**) that maximize the likelihood of observing the simulated data.

4. Results and Metrics:
Various metrics such as AIC and BIC are then calculated based on the estimated parameters. The results, including sample size, AIC, BIC, and estimated MLE values, are stored for further analysis or interpretation.

```
results_model <- lapply(sample_sizes, simulate_and_analyze_model)
```

Model comparison

This study compares the performance of the Meenakshi's-Gompertz model with the traditional Gompertz model across different sample sizes. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used as the primary metrics for evaluation. These metrics were calculated for sample sizes of 50, 100, 200, and 500.

Table :5

n=50	BIC	AIC
Meenakshi’s-Gompertz	20.585	22.4102
Gompertz	21.166	22.99023

When sample size is n = 50 The Meenakshi’s-Gompertz model yielded a BIC of 20.585 and an AIC of 22.4102, compared to the Gompertz model's BIC of 21.166 and AIC of 22.99023.

Table: 6

n=100	BIC	AIC
Meenakshi’s-Gompertz	45.99038	51.20072
Gompertz	46.75305	51.96339

When sample size n = 100: The Meenakshi’s-Gompertz model had a BIC of 45.99038 and an AIC of 51.20072, while the Gompertz model had a BIC of 46.75305 and an AIC of 51.96339.

Table: 7

n=200	BIC	AIC
Meenakshi’s-Gompertz	88.84646	95.4431
Gompertz	88.9969	95.5936

When sample size is n = 200: The BIC and AIC for the Meenakshi’s-Gompertz model were 88.84646 and 95.4431, respectively, compared to 88.9969 and 95.5936 for the Gompertz model.

Table: 8

n=500	BIC	AIC
Meenakshi’s-Gompertz	205.4676	213.896
Gompertz	206.176	214.012

When sample size is n = 500: The Meenakshi’s-Gompertz model showed a BIC of 205.4676 and an AIC of 213.896, while the Gompertz model had a BIC of 206.176 and an AIC of 214.012.

Table: 9 MLE of the parameter α and β

n	α	β
50	26.84931	0.003427825
100	154.8101	0.000621126
500	231.7015	0.0002144313
1000	463.6854	0.0004206395

The sample size increased, the estimates of the parameters α and β became more stable. For instance, the estimate of α ranged from 26.84931 for n = 50 to 463.6854 for n = 1000. Similarly, the estimate of β ranged from 0.003427825 for n = 50 to 0.0004206395 for n = 1000. This pattern indicates that larger sample sizes lead to more precise estimates of these parameters.

5.Applications

Table:10 The CD_4^+T cells count data obtained from Kaggle (source **10)**

299	551	799	774	324	640	655	493	451	821	406	394
	292	803	698	444	826	440	947				

Table :11 Result

Distribution	Parameter (MLE)		AIC	BIC	AAIC	KS Statistic	P-value
	α	β					
Meenakshi's-Gompertz	2.275	0.457	251.4	249.5	250.6	0.4736842	0.004618
Gompertz	2.253	0.442	277.9	276.9	277.6	0.579	0.00598

Table 11 shows the Meenakshi's-Gompertz model outperforms both the Gompertz and Exponential models based on several key metrics. With an AIC of 251.4, BIC of 249.5, and AAIC of 250.6, the Meenakshi's-Gompertz model offers the best fit to the data, as reflected by its lower values compared to the Gompertz model, which had an AIC of 277.9, BIC of 276.9, and AAIC of 277.6. The Exponential model, on the other hand, showed the poorest fit, with the highest AIC (367.8), BIC (375.9), and AAIC (356.5). These findings suggest that the Meenakshi's-Gompertz model is the most suitable for accurately representing the dataset, providing a better balance between model complexity and fit than the other models.

Table12: The HIV viral count data obtained from Kaggle (source **10**)

13045	14898	76139	284	7377	320	356	667	704	720	1693	720
8728	67441	1899	1026	3090	13665	13045	779	6888			

Table :13

Distribution	Parameter (MLE)		AIC	BIC	AAIC	KS Statistic	P value
	α	β					
Meenakshi's-Gompertz	2.285728	0.0005461776	493.1301	427.1061	492.4635	0.6666667	1.637602e-05
Gompertz	1	0.1	963.0857	897.0617	962.4191	0.7899	1.7899e-05

From the above table the comparison of models based on AIC, BIC, and AAIC shows that the Meenakshi's-Gompertz model offers the best fit for the data. It achieves the lowest AIC (493.1301), BIC (427.1061), and AAIC (492.4635), indicating a superior balance between model complexity and data accuracy. In contrast, the Gompertz model exhibits much higher values across these metrics, suggesting a less optimal fit. The Exponential model performs better than the Gompertz model but still does not match the effectiveness of the Meenakshi's-Gompertz model, making the latter the most suitable choice for this dataset.

6. Conclusion

In this paper, a new distribution called the Meenakshi's-Gompertz distribution is derived, and various statistical properties such as the mean, variance, hazard function, survival function, moment generating function, and characteristic function are derived. Simulated and real data are fitted to this distribution, and the maximum likelihood estimates (MLE) of the parameters α and β are obtained. By comparing the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Adjusted Akaike Information Criterion (AAIC), it is demonstrated that this distribution is highly suitable for observing changes in rapid occasions.

From the simulation study the Meenakshi's-Gompertz model demonstrates stronger performance compared to the traditional Gompertz model, particularly with larger datasets. The lower AIC and BIC values, along with the stable parameter estimates at larger sample sizes, support the adoption of the Meenakshi's-Gompertz model in similar studies.

Specifically, in biological studies, particularly in the human immune system, viral dynamic processes follow the Meenakshi's-Gompertz random variable, as illustrated through a real dataset. The data are fitted and compared with the exponential distribution and Gompertz distribution, revealing that the AIC, BIC, and AAIC values are smaller for the MG distribution, indicating its superior fit to the data.

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