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Generalized Estimator of Population Variance utilizing Auxiliary Information in Simple Random Sampling Scheme

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ABSTRACT

In this study, using the Simple Random Sampling without Replacement (SRSWOR) method, we propose a generalized estimator of population variance of the primary variable. Up to the first order of approximation, the bias and Mean Squared Error (MSE) expressions for the suggested estimator are produced. The suggested estimator's characterizing scalar is optimized, and for this optimal value of the characterizing constant, the suggested estimator's least MSE is also determined. The efficiency criteria of the suggested estimator over the other estimators are determined after a theoretical comparison of the proposed estimator with the other population variance estimators that already exist. Several actual natural populations are used to validate these efficiency parameters. For practical use in various application domains, the estimator with the lowest MSE and the best Percentage Relative Efficiency (PRE) is advised.

Keywords: Main variable, Auxiliary variable, Estimator, Bias, MSE, PRE.

1. Introduction

In the survey sampling theory, calculating the population parameter under consideration is always preferred, but when the population is very large, it is an expensive and time-consuming task to collect information on every unit of the population. As a result, the alternative to it is sampling from the targeted population. The parameter under consideration is estimated through an estimator using the sample information. The related statistics are the most appropriate estimators for the parameter under discussion, making sample variance the best estimator for population variance of the study variable. One of the major drawbacks of the sample variance estimator is that it has a reasonably large sampling variance. Now we seek even for a biased estimator of population variance but having lesser MSE as compared to sampling variance of sample variance. The right use of an auxiliary variable, which has a strong positive or negative correlation with the primary variable under research, achieves this goal. In terms of its parameters, estimators using auxiliary information are more effective than those, using sample variance. When the main variable Y and the auxiliary variable X are significantly positively linked and the line of regression of Y on X crosses through the origin, the ratio type estimators under the ratio method of estimation are employed to improve estimation of population variance σ_y^2 of the main variable Y. The product type estimators under the product method of estimation are used for enhanced estimation of σ_y^2 when the variable Y and X has high degree negative correlation and the line of regression of Y on X crosses through origin. On the other hand if the regression line of Y on X does not pass cross the origin or its neighborhood, regression estimators under the regression method of estimation are used for elevated estimation of σ_y^2.

The right application of auxiliary information during the estimating stage improves the effectiveness of the estimator under discussion, as is well established in the literature. Using the

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auxiliary data, Singh and Singh (2001) proposed a ratio-type estimator for a better estimation of the population variance. A better regression method for assessing population variance in a two-phase sample design was later developed by Singh and Singh (2003). Jhajj *et al.* (2005) have introduced a valuable family of chain estimators for the elevated estimation of the population variance using the sub-sampling method. Furthermore, the creation of auxiliary parameter-based variance estimation was the main emphasis of Shabbir and Gupta (2007) study. The variance estimation of the simple random sampling strategy was then improved upon by a number of methods proposed by Kadilar and Cingi (2007). Using information on the kurtosis of an auxiliary variable in sample surveys, Singh *et al.* (2008) created a ratio and product type estimator that is asymptotically unbiased and estimates the limited population variance. Regarding the improved estimation of σ_y^2 using auxiliary parameters, Grover (2010) published a correction. Singh and Solanki (2012) also suggested a novel technique for variance estimation in simple random sampling that makes use of auxiliary data.

On the other hand, Yadav and Kadilar (2014) proposed a two-parameter enhanced σ_y^2 estimator using auxiliary parameters. In order to estimate σ_y^2 using auxiliary variable quartiles, Singh and Pal (2016) proposed an improved family of estimators. Yadav *et al.* (2017) developed a better σ_y^2 estimator using the auxiliary variable's tri-mean and inter-quartile range, which are known quantities. A better estimate of σ_y^2 utilizing the well-known tri-mean and third quartile of the auxiliary variable has been proposed by Yadav *et al.* (2019). Naz *et al.* (2020) used unconventional dispersion measures of X , which had a strong connection with Y under consideration, and presented ratio-type estimators of σ_y^2 when outliers were present. A novel exponential ratio estimate of population variance was developed by Olayiwola *et al.* (2021), and it has demonstrated improvements over many other existing estimators of σ_y^2. Utilizing the well-known auxiliary parameters, Bhushan *et al.* (2022) proposed several new modified classes of population variance. Ahmed and Hussein (2022) worked on some ratio estimators of σ_y^2 utilizing the known auxiliary parameters in ranked set sampling scheme. Sharma *et al.* (2022) suggested an improved variance estimator utilizing known auxiliary parameters for decision-making.

In this paper, we suggest a general ratio type estimator of σ_y^2 and make use of a known auxiliary parameters in order to improve the σ_y^2 estimation of Y. The MSE and bias are both assessed up to an approximation of order one. Sections have been created for the remaining segments of the paper. Section 2 provides a review of σ_y^2 estimators for the research variable using known auxiliary variable parameters. Section 3 describes the suggested estimators and their sample characteristics up to the first order approximation. In Section 4, the criteria for the superiority proposed estimator over competing estimators are described, along with an efficiency comparison of the introduced estimator with the competing estimators. The biases and MSEs for the actual natural populations were calculated in the empirical investigation described in Section 5. In Section 6, the conclusions from the results of the numerical investigation are discussed. The conclusion and findings of the study are reported in Section 7, and the document concludes with a list of references.

2. Review of Existing Estimators

In this section, different estimators of population variance σ_y^2 of Y using and without using auxiliary parameters are presented in Table 1 with their MSEs and/or sampling variance for an approximation of degree one.

Table 1. Different estimators and their variance/Mean Squared Errors

S.No.	Estimator with Authors	Variance/MSE
1.	$t_0 = s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ Sample Variance Estimator	$V(t_0) = \gamma S_y^4 (\lambda_{40} - 1)$
2.	$t_r = s_y^2\left[\frac{S_x^2}{s_x^2}\right]$ Isaki (1983) Estimator	$MSE(t_r) = \gamma S_y^4 [(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)]$
3.	$t_1 = s_y^2\left[\frac{S_x^2 - \beta_2}{s_x^2 - \beta_2}\right]$ Kadilar and Cingi (2005)	$MSE(t_1) = \gamma S_y^4 [(\lambda_{40} - 1) + R_1^2(\lambda_{04} - 1) - 2R_1(\lambda_{22} - 1)]$
4.	$t_2 = s_y^2\left[\frac{S_x^2 C_x - \beta_2}{s_x^2 C_x - \beta_2}\right]$ Kadilar and Cingi (2005)	$MSE(t_2) = \gamma S_y^4 [(\lambda_{40} - 1) + R_2^2(\lambda_{04} - 1) - 2R_2(\lambda_{22} - 1)]$
5.	$t_3 = s_y^2\left[\frac{S_x^2 - C_x}{s_x^2 - C_x}\right]$ Kadilar and Cingi (2005)	$MSE(t_3) = \gamma S_y^4 [(\lambda_{40} - 1) + R_3^2(\lambda_{04} - 1) - 2R_3(\lambda_{22} - 1)]$
6.	$t_4 = s_y^2\left[\frac{S_x^2 \beta_2 - C_x}{s_x^2 \beta_2 - C_x}\right]$ Kadilar and Cingi (2005)	$MSE(t_4) = \gamma S_y^4 [(\lambda_{40} - 1) + R_4^2(\lambda_{04} - 1) - 2R_4(\lambda_{22} - 1)]$
7.	$t_5 = s_y^2\left[\frac{S_x^2 + Q_1}{s_x^2 + Q_1}\right]$ Subramani and Kumarapandiyan (2012)	$MSE(t_5) = \gamma S_y^4 [(\lambda_{40} - 1) + R_5^2(\lambda_{04} - 1) - 2R_5(\lambda_{22} - 1)]$
8.	$t_6 = s_y^2\left[\frac{S_x^2 + Q_3}{s_x^2 + Q_3}\right]$ Subramani and Kumarapandiyan (2012)	$MSE(t_6) = \gamma S_y^4 [(\lambda_{40} - 1) + R_6^2(\lambda_{04} - 1) - 2R_6(\lambda_{22} - 1)]$
9.	$t_7 = s_y^2\left[\frac{S_x^2 + Q_R}{s_x^2 + Q_R}\right]$ Subramani and Kumarapandiyan (2012)	$MSE(t_7) = \gamma S_y^4 [(\lambda_{40} - 1) + R_7^2(\lambda_{04} - 1) - 2R_7(\lambda_{22} - 1)]$
10.	$t_8 = s_y^2\left[\frac{S_x^2 + Q_D}{s_x^2 + Q_D}\right]$	$MSE(t_8) = \gamma S_y^4 [(\lambda_{40} - 1) + R_8^2(\lambda_{04} - 1) - 2R_8(\lambda_{22} - 1)]$

	Subramani and Kumarapandiyan (2012)	
11.	$t_9 = s_y^2\left[\frac{S_x^2+Q_A}{s_x^2+Q_A}\right]$ Subramani and Kumarapandiyan (2012)	$MSE(t_9)=\gamma S_y^4[(\lambda_{40}-1)+R_9^2(\lambda_{04}-1)-2R_9(\lambda_{22}-1)]$
12.	$t_{10} = s_y^2\left[\frac{S_x^2+S_x\beta_2}{s_x^2+S_x\beta_2}\right]$ Khan *et al.* (2020)	$MSE(t_{10})=\gamma S_y^4[(\lambda_{40}-1)+R_{10}^2(\lambda_{04}-1)-2R_{10}(\lambda_{22}-1)]$

where,

$$\gamma=\frac{1}{n}-\frac{1}{N}\ ,\quad S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2\ ,\quad \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i\ ,\quad \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i\ ,\quad \lambda_{rs}=\frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}\ ,$$

$$\mu_{rs}=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^r(X_i-\bar{X})^s\ ,\ S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{X})^2\ ,\ R_1=\frac{S_x^2}{S_x^2-\beta_2}\ ,\ R_2=\frac{S_x^2C_x}{S_x^2C_x-\beta_2}\ ,$$

$$R_3=\frac{S_x^2}{S_x^2-C_x}\ ,\ R_4=\frac{S_x^2\beta_2}{S_x^2\beta_2-C_x}\ ,\ R_5=\frac{S_x^2}{S_x^2+Q_1}\ ,\ R_6=\frac{S_x^2}{S_x^2+Q_3}\ ,\ R_7=\frac{S_x^2}{S_x^2+Q_R}\ ,\ R_8=\frac{S_x^2}{S_x^2+Q_D}\ ,$$

$$R_9=\frac{S_x^2}{S_x^2+Q_A}\ ,\ R_{10}=\frac{S_x^2}{S_x^2+S_x\beta_2}$$

3. Proposed Estimator

Motivated by various authors in the literature, we suggest a generalized estimator of σ_y^2 using the Khan *et al.* (2020) estimator as,

$$t_p = s_y^2\left[\frac{S_x^2+S_x\beta_2}{s_x^2+S_x\beta_2}\right]^{\delta} \tag{1}$$

where, δ is the characterizing constant to be determined such that the MSE of t_p is minimum.

To study the bias and MSE of the suggested estimator, we use the following standard approximations as,

$s_y^2 = S_y^2(1+\varepsilon_0)$ and $s_x^2 = S_x^2(1+\varepsilon_1)$ such that $E(\varepsilon_i)=0$ for $(i=0,1)$ and $E(\varepsilon_0^2)=\gamma(\lambda_{40}-1)$, $E(\varepsilon_1^2)=\gamma(\lambda_{04}-1)$, $E(\varepsilon_0\varepsilon_1)=\gamma(\lambda_{22}-1)$.

The proposed estimator t_p in (1) may be expressed in terms of ε_i's as,

$$t_p = S_y^2(1+\varepsilon_0)\left[\frac{S_x^2+S_x\beta_2}{S_x^2(1+\varepsilon_1)+S_x\beta_2}\right]^{\delta}$$

$$= S_y^2(1+\varepsilon_0)\left[\frac{S_x^2 + S_x\beta_2}{S_x^2 + S_x\beta_2 + S_x^2\varepsilon_1}\right]^{\delta}$$

$$= S_y^2(1+\varepsilon_0)\left[\frac{1}{1+\dfrac{S_x^2}{S_x^2 + S_x\beta_2}\varepsilon_1}\right]^{\delta}$$

$$= S_y^2(1+\varepsilon_0)\left[\frac{1}{1+R_{10}\varepsilon_1}\right]^{\delta}$$

$$= S_y^2(1+\varepsilon_0)(1+R_{10}\varepsilon_1)^{-\delta}$$

$$= S_y^2(1+\varepsilon_0)[1-\delta R_{10}\varepsilon_1 + \delta(\delta+1)R_{10}^2\varepsilon_1^2]$$

$$= S_y^2[1+\varepsilon_0 - \delta R_{10}\varepsilon_1 - \delta R_{10}\varepsilon_0\varepsilon_1 + \delta(\delta+1)R_{10}^2\varepsilon_1^2]$$

Subtracting S_y^2 on both sides of above equation, we have,

$$t_p - S_y^2 = S_y^2[\varepsilon_0 - \delta R_{10}\varepsilon_1 - \delta R_{10}\varepsilon_0\varepsilon_1 + \delta(\delta+1)R_{10}^2\varepsilon_1^2] \qquad (2)$$

Using the values of various expectations and taking into account both sides of (2), we get the bias of t_p for an approximation of order one as,

$$B(t_p) = \gamma S_y^2[\delta(\delta+1)R_{10}^2(\lambda_{04}-1) - \delta R_{10}(\lambda_{22}-1)] \qquad (3)$$

For an approximation of order one, from equation (2), we have,

$$t_p - S_y^2 \cong S_y^2[\varepsilon_0 - \delta R_{10}\varepsilon_1]$$

Squaring on both sides of above equation, we have

$$(t_p - S_y^2)^2 = S_y^4[\varepsilon_0^2 - \delta^2 R_{10}^2\varepsilon_1^2 - 2\delta R_{10}\varepsilon_0\varepsilon_1]$$

Setting values of various expectations and taking into account the expectations on both sides of the aforementioned equation, we get the MSE of t_p as,

$$MSE(t_p) = \gamma S_y^4[(\lambda_{40}-1) - \delta^2 R_{10}^2(\lambda_{04}-1) - 2\delta R_{10}(\lambda_{22}-1)] \qquad (4)$$

The best value of δ is obtained by differentiating equation (4) with regard to δ and equating it to zero as,

$\dfrac{\partial}{\partial\delta}MSE(t_p) = 0$, we have,

$$\delta_{opt} = \frac{(\lambda_{22}-1)}{R_{10}(\lambda_{04}-1)} \qquad (5)$$

Putting the optimum value of δ in equation (5) in the equation (4), we get the minimum value of MSE of t_p as,

$$MSE_{\min}(t_p) = \gamma S_y^4\left[(\lambda_{40}-1) - \frac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}\right] \qquad (6)$$

4. Efficiency Comparison

The proposed estimator has been contrasted with the previously discussed competing estimators of population variance in this section. The efficiency requirements that must be met for the suggested estimator to outperform the rival estimators are also determined and are shown in Table 2. The introduced estimator t_p is more efficient than the competing estimators $t_i\,(i=0,r,...,10)$ under the condition if, $MSE(t_i)-MSE_{\min}(t_p)>0$.

Table 2. Efficiency conditions of proposed estimator over competing estimators

Estimator	Efficiency condition
t_0	$(\lambda_{22}-1)^2>0$
t_r	$(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2(\lambda_{22}-1)$
t_1	$R_1^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_1(\lambda_{22}-1)$
t_2	$R_2^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_2(\lambda_{22}-1)$
t_3	$R_3^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_3(\lambda_{22}-1)$
t_4	$R_4^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_4(\lambda_{22}-1)$
t_5	$R_5^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_5(\lambda_{22}-1)$
t_6	$R_6^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_6(\lambda_{22}-1)$
t_7	$R_7^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_7(\lambda_{22}-1)$
t_8	$R_8^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_8(\lambda_{22}-1)$
t_9	$R_9^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_9(\lambda_{22}-1)$
t_{10}	$R_{10}^2(\lambda_{04}-1)+\dfrac{(\lambda_{22}-1)^2}{(\lambda_{04}-1)}>2R_{10}(\lambda_{22}-1)$

5. Numerical Study

We have taken into account two natural populations in this section in order to confirm the efficiency criteria of the suggested estimator over the competing estimators. The main and the auxiliary variables from two natural populations are as follows:

Population-1 [Source: Gupta and Shabbir (2008)]

Y : Number of agricultural labourers for the year 1971

X : Number of agricultural labourers for the year 1961

Population-2 [Source: Kadilar and Cingi (2005)]

Y : Production level of apples per 100 tons

X : Number of trees (1 unit shows 100 apple trees)

The parameters of both real populations are presented in Table-3.

Table 3. Population parameters of two real has

Population-I	Population-II
$N = 278$, $n = 30$, $\bar{Y} = 39.068$, $\bar{X} = 25.111$, $S_y = 56.457$, $S_x = 40.674$, $\lambda_{40} = 25.896$, $\lambda_{04} = 38.889$, $\lambda_{22} = 26.814$, $\rho = 0.721$	$N = 173$, $n = 20$, $\bar{Y} = 4.04$, $\bar{X} = 98.44$, $S_y = 9.46$, $S_x = 187.94$, $\lambda_{40} = 27.96$, $\lambda_{04} = 28.10$, $\lambda_{22} = 23.08$, $\rho = 0.891$

The MSE of different competing estimators and the proposed estimator and the PRE of different estimators with respect to t_0 are presented in Table-4:

Table 4: MSE of different estimators and PRE with respect to t_0

Population-I			Population-II		
Estimator	**MSE**	**PRE**	**Estimator**	**MSE**	**PRE**
t_0	7521187.116	100.000	t_0	9547.72201	100.000
t_r	3778793.028	199.037	t_r	3506.02551	272.323
t_1	3983111.977	188.827	t_1	3511.14624	271.926
t_2	4862885.344	154.665	t_2	4862.88534	196.339
t_3	4862205.422	154.687	t_3	4862.20542	196.366
t_4	4862885.467	154.665	t_4	4862.88546	196.339

t_5	3468925.171	216.816	t_5	3504.57264	272.436
t_6	3022246.855	248.861	t_6	3502.82491	272.572
t_7	3220379.202	233.551	t_7	3504.16751	272.468
t_8	3468925.171	216.816	t_8	3506.54872	272.283
t_9	3220379.202	233.551	t_9	3503.61287	272.511
t_{10}	2846652.986	264.212	t_{10}	3497.55713	272.983
t_p	2208010.086	340.632	t_p	3176.70558	300.554

The following Figure-1 and Figure-2 show the MSEs of different estimators for Population-I and Population-II respectively.

Figure-1: MSE of different estimators for Population-I

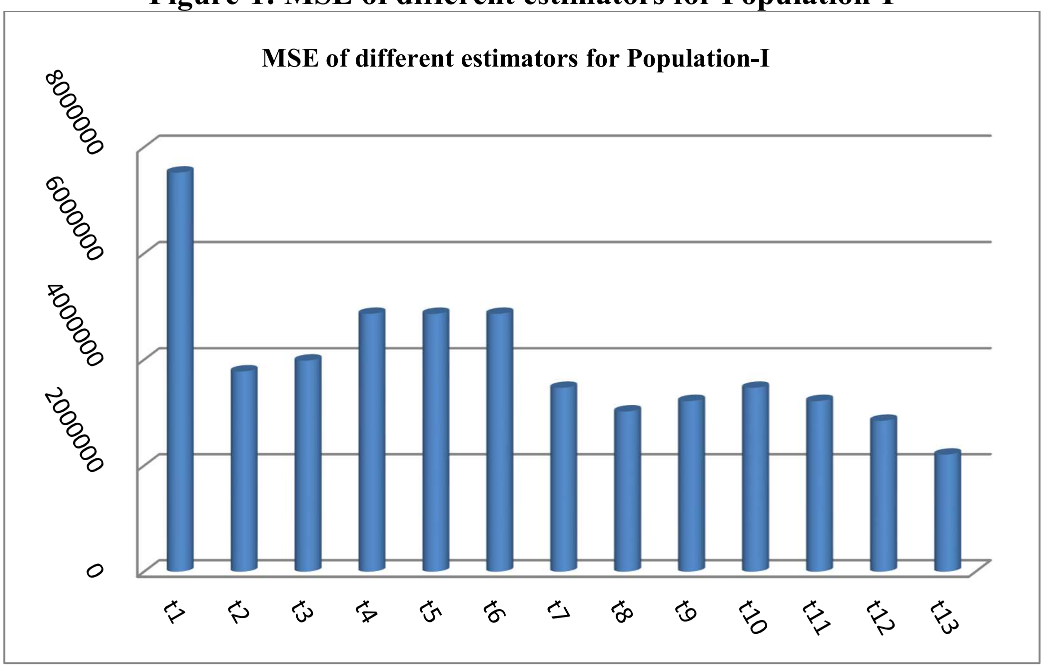
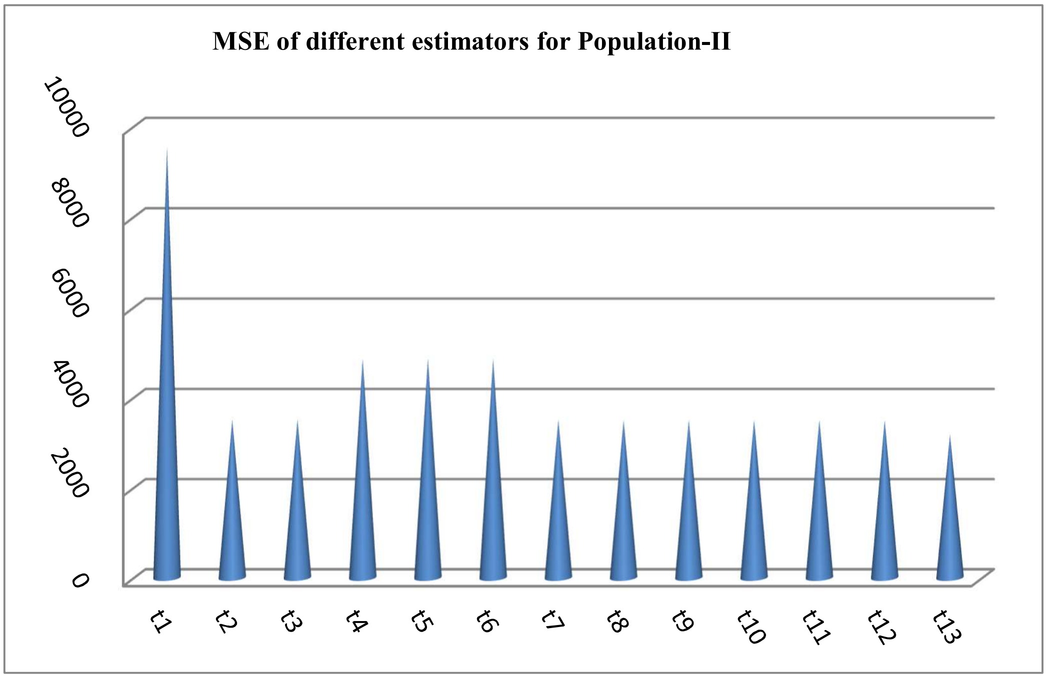


Figure-2: MSE of different estimators for Population-II



6. Results and Conclusion

In this paper, we propose a known auxiliary parameters-based generalized ratio type estimator of population variance. For the first order of approximation, the proposed estimator's sampling characteristics, bias, and MSE are determined. The proposed estimator's characterizing constant has been given the ideal value. For this ideal value of the characterizing constant, the MSE of the proposed estimator's minimal value has also been determined. The proposed estimator is contrasted with the current rival estimators both conceptually and experimentally. From Table 4, it may be observed that the proposed estimator t_p have the least MSE for both the populations among the all competing estimators, t_0 , t_r , t_1 - t_{10} , which may also been verified through the graphs in Figure-1 and Figure-2 respectively. The suggested estimator is therefore the most effective estimator among those used by Khan *et al.* (2020), Kadilar and Cingi (2005), Isaki (1983) ratio estimator, sample variance estimator, Subramani and Kumarapandiyan (2012), and Kadilar and Cingi (2005). Therefore, it is advised that the proposed estimator be used practically in many application areas.

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