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NEW SERIES OF SUPER-SATURATED DESIGN

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ABSTRACT

In this paper, a new series supersaturated design is proposed using Partially Balanced Incomplete Block Design through a combinatorial arrangement of the incidence matrix of a Balanced Incomplete Block Design. The method was illustrated with suitable example.

Keywords: SSD, incidence matrix, BIBD.

1. INTRODUCTION

A design matrix 'X' is said to be saturated if the number of design points is equal to the number of factors plus one and is the design matrix 'X' is said to be a super-saturated, if the number of factors is more than the number of design points.

Super-saturated design is a fractional factorial design and the degrees of freedom for all its main effects and interaction terms exceed the number of design points. These designs reduce the experimental cost and time significantly due to a smaller number of experimental runs. These designs used to identify active factor main effects when experimentation is expensive and the number of potential factors is large. These designs are more economical and flexible due to their run size. Satterthwaite (1959) initially made an attempt to construct saturated designs randomly and suggested the random balance designs. Booth and Cox (1962) proposed a systematic method for the construction of super-saturated designs.

2. CONSTRUCTION OF A NEW SERIES OF SUPER-SATURATED DESIGN

In this section, a new series of super-saturated design was constructed using a Balanced Incomplete Block Design (BIBD) and the method is illustrated with a suitable example.

Theorem: A super-saturated design X with 2v experimental runs in 2b factors with two levels can be constructed using the combinatorial arrangement of the incidence matrix N of a Balanced Incomplete Block Design (BIBD) with parameters v, b (≠v), r, k, λ and its dual $\overline{N}$ in the form $\begin{bmatrix} N & \overline{N} \\ \overline{N} & N \end{bmatrix}$.

Method of Construction:

Step-1: Let N be the incidence matrix of a BIBD with parameters v, b (≠v), r, k, λ.

Step-2: Replace the elements 0's with 1's and 1's with 0's, i.e. dual of N be $\overline{N}$.

Step -3: Arrange the incidence matrix N and its dual $\overline{N}$ in the following form

$$\begin{bmatrix} N & \overline{N} \\ \overline{N} & N \end{bmatrix}.$$

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Step-4: Replace 0 element with -1's in the above combinatorial arrangement to obtain a super saturated design X for V factors in B experimental runs, where V = 2b and B= 2v.

EXAMPLE 2.1: Consider a Balanced Incomplete Block Design with parameters v = 5, b =10, r = 6, k = 3, λ = 3 whose incidence matrix is N. where N and $\bar{N}$ are

$$N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad \bar{N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The combinatorial arrangement of N and $\bar{N}$ in N′ provides an incomplete block design with parameters v′ = 10, b′ = 20, r′ = 10, k′ = 5 with an incidence matrix

$$\begin{bmatrix} N & \bar{N} \\ \bar{N} & N \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

By replacing 0's with -1's, treating each column related to a factor and rows as experimental runs provides a super-saturated design X with 10 experimental runs in 20 factors. The resulting super saturated design X_{10x20} is

$$\mathbf{X} = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 \\ -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 \end{bmatrix}$$

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