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Estimation of Population Size Using Ranked Set Sampling and Some of its Variations

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ABSTRACT
The purpose of this paper is to estimate the population size of known total, using a sample that chosen using ranked set sampling technique and some of its variations; in particular, Ranked Set Sampling (RSS), Moving Extreme Ranked Set Sampling (MERSS) and Median Ranked Set(MRSS) are considered. The estimators obtained are compared with their counterparts using simple random sampling(SRS). It turned out that the estimators using RSS and its variations are more efficient than the corresponding estimators using SRS.

Keywords: Population size, Simple random sampling; Ranked set sampling; Moving extreme ranked set sampling; Median ranked set sampling.

1. Introduction

Statistics is the science that deals with the understanding of the population characteristics based on a proper portion of it. More precisely, statistics is the science of collecting information from a suitable portion of the population of interest (a representative sample), properly organizing and summarizing the collected information, making inferences about the population, and providing the accuracy of these inferences.

One main branch of inferential statistics is the estimation of some of the population parameters such as the mean(μ), variance(σ^2), total (τ) proportion(p), etc. The size of the population (*N*) is usually known, but sometimes, it is unknown and need to be estimated. For examples, the size of "*Fish population*" in a sea "*Beggars population*" in a city, "the total number of Oranges" in a truckload of oranges, etc. Most of the time, it is very hard to count the number of elements in **mobile populations** and in a population which consists of very similar elements that are hard to distinguish among them (Fish, Camels, etc.). We should thank **ALLAH** for being of different voices and shapes:

قال تعالى: "وَمِنْ آيَاتِهِ خَلْقُ السَّمَاوَاتِ وَالْأَرْضِ وَاخْتِلَافُ أَلْسِنَتِكُمْ وَأَلْوَانِكُمْ "ۚ

Allah said in the ***Holly Quran****:*

"One of Allah Verses is the creation of the Heavens & the Earth and the difference of our Tongues & Colors"

There are several techniques for the estimation of the population size and population total. In this work, the goal is the estimation of N for a population of known total τ based on a random sample obtained using ranked set sampling technique or some of its variations.

The next section contains a short description of the main sampling techniques and a popular method of the estimation of N and τ. The literature review of this topic and a summary of the proposed work is the content of Section 3. The previous work related

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to these topics is given in section 3, the estimation of N using SRS and RSS is considered in section 4, the use of MERSS to estimate N is the content of section 5, MRSS is considered section 6. The suggested estimators are compared to the corresponding estimators using SRS. Concluding remarks are given in section 7.

2. Sampling Techniques and Methods of Estimation of the Population Size and Total

The accuracy of the inference depends mainly on the **sample size** and the **sampling technique** used to choose the sample. The main sampling techniques are Simple, Stratified, Cluster and Systematic random sampling. A sample of size n from a population of size N is called a ***simple random sample(SRS)*** if it is chosen so that *all possible samples of size n have equal chances of being chosen*; i.e. the probability for any sample of size n to be the chosen sample is $1/\binom{N}{n}$. As a consequence of this definition, the probability that any element of the population will be in the chosen SRS (Inclusion Probability) is: $\binom{N-1}{n-1}/\binom{N}{n}=\frac{n}{N}$.

In **Stratified Random Sampling** technique, the population is partitioned (stratified) into L groups called "strata". The division is appropriate if the elements of the groups are *similar within* (measured by variance) and *different among* (measured by coefficient of variation). A stratified sample consists of L random samples, one from each stratum.

For **Cluster Random Sampling** technique, the population consists of N groups called "clusters". A cluster sample is a random sample of n clusters. For cluster sampling to be effective, the elements of the clusters should be *different within* and *similar among*.

To obtain a random sample using the above techniques, a frame of the population should be available (a list of all elements of the population). It happens in practice that the frame is not available before taking the sample. To obtain a systematic random sample, we choose one element at random from the first k elements and include in the sample every k^{th} element thereafter. The obtained sample is called $1-in-k$ systematic sample.

For more details about the above 4 sampling techniques, see [23] & [16]**.**

Ranked Set Sampling(RSS) is a newer sampling method. It was suggested by [18] for estimating the pasture yields. A ranked set sample can be obtained using the following steps:

Step (1): Choose a random sample of size m from the population.

Step (2): Order by **judgment** the elements of the chosen random sample with respect to the variable of interest and choose the element with minimum value for actual measurement, i.e., $X_{(1:m)}$.

Step (3): Repeat steps (1) & (2), but choose the second minimum, $X_{(2:m)}$ for actual measurement.

Step (4): The above process continues until obtaining the maximum $X_{(m:m)}$ for actual measurement.

Step (5): Steps (1-4) may be repeated r times if needed, to obtain a sample of size $n = rm$.

To avoid ranking error, m (set size) should be small; the size of the sample can be increased by increasing r not m . The elements of RSS can be denoted by:

$X_{(i:m)}^{j} : i = 1,2,...,m,\ j = 1,2,...r$, $X_{(i:m)}^{j}$ is the i^{th} order statistic of the i^{th} random sample of size m at the j^{th} cycle, assuming that judgment ranking is the same as actual ranking (no error in ranking), **Clearly a SRS consists of m dependent order statistics, while a RSS of size m (** For one cycle) **consists of m independent order statistics.**

There are many variations of RSS. Double RSS (DRSS) is a variation of RSS introduced by [10]. It is simply a RSS obtained based on m RSSs. Multistage RSS (MSRSS), introduced by [11], is generalization of DRSS. If the number of stages in MSRSS goes to infinity, then the sample is called a Steady-State Ranked Set Sampling (SSRSS). It was shown by [6] that Steady-State Ranked Set Sampling is very similar to Stratified random sampling. Median ranked set sampling(MRSS) was introduced by [19].

Moving Extreme RSS (MERSS) was first introduced by [4] and coined MERSS by [8]. It was described as follows:

Step (1): Select m random samples of size 1, 2,…, m .

Step (2): Order the elements by judgment, without actual measurement of the variable of interest.

Step (3): Measure accurately the judgment maximum ordered observation from the first set $X_{(1:1)}$, the judgment maximum ordered observation from the second set $X_{(2:2)}$. The process continues in this way until the judgment maximum ordered observation from the last m^{th} sample is measured, $X_{(m:m)}$.

Step (4): Steps (1-3) may be repeated if necessary on m samples of size 1,2,…, m , respectively, but the judgment minimum ordered observation is measured instead of the judgment maximum.

Methods of Estimation of the Population Size and Total

The well-known techniques of estimating the size and the population total are outlined below:

(1) Ratio Estimation

Ratio estimation is a technique used to estimate τ for the variable of interest using an auxiliary variable. Let μ_y be the mean of the variable of interest, which can be estimated by the sample average, $\overline{Y}$. Let τ_y be the population total, which is $N\mu_y$. It can be estimated by $N\overline{Y}$ if N is known. If N is unknown and τ_x (assumed known), is the total of the values of the auxiliary variable X then the population ratio (R) is :

$$R = \frac{\tau_y}{\tau_x} = \frac{N\mu_y}{N\mu_x} = \frac{\mu_y}{\mu_x}. \text{ Thus, } \tau_y = \frac{\mu_y}{\mu_x}\tau_x .$$

If $(X_1,Y_1),(X_2,Y_2),\ldots,(X_n,Y_n)$ is a SRS, then

$$\hat{\tau}_y = \frac{\overline{Y}}{\overline{X}}\tau_x \ \& \ \hat{R} = r = \frac{\overline{Y}}{\overline{X}} .$$

For more details, see [22] and [1].

(2) Capture-Recapture Method:
This method was used for the first time to estimate the size of French population by Laplace in 1786 and it was designed by Lincolon-Petersen in 1894; see [24]. There are two types of Capture-Recapture Technique: Capture-Recapture Technique Using Direct Sampling Capture-Recapture Technique Using Inverse Sampling. This method using RSS will be considered in a future work.

3. Literature Review

Estimation of Population Size and Total

[13] considered the estimation of the size of a closed population when capture probabilities vary among animals. [22] applied the capture recapture method to estimate the death and injury rates due to road traffic accidents in Karachi, Pakistan. [20] considered the estimation of the size of a closed population using stratified sampling. [2] considered the estimation of the population total when the population size is unknown. [25] estimated the number of people eligible for health service. [12] considered the estimation of the number of drug users in Bangkok using capture recapture method. [15] used capture-recapture method to estimate the size of the eastern Canada -West Greenland Bowhead "Whale population". [3] considered the estimation of the Population Total-Utilizing Estimators of the Population size. [17] investigated ratio-type estimators of the population mean μ_y of the study variable Y, involving either the first or the third quartile of the auxiliary variable X, using RSS and ERSS methods.

Ranked Set Sampling and Some of its Variations

[18] presented the first paper on RSS where he estimated the average yield of pastures. [26] established the theory of RSS. An annotated bibliography of literature on RSS until 1995 was provided by [14].
[8] introduced the double RSS procedure (DRSS) for estimating the population mean. [4] introduced the moving extreme ranked set sampling (MERSS). [11] introduced the multistage ranked set sampling (MSRSS) as a generalization of the double RSS. [8] investigated MERSS and used it to estimate the mean of the exponential distribution. [9] estimated the location parameter of symmetric distribution using MERSS. For more details about RSS and its variation, see [5]. [7] studied the accuracy of ranking in moving extreme ranked set sampling.

4. Estimation of the Population Size Using Ranked Set Sampling Technique

In this section, we will consider the estimation of the population size, N , using ranked set sampling technique. The suggested estimators will be compared to their counter parts using simple random sampling technique.

Estimation the Population Size Using SRS

We want to estimate the population size N assuming that the population total of the variable of interest, τ_X, is known. For example, we can easily find the total weight of oranges in a truckload, but it is not easy to count the number of oranges. For completeness, we first review the procedure of estimation using the main sampling technique, SRS. Now,

$\tau_X = N\mu \rightarrow N = \frac{\tau_X}{\mu}$. Thus, to estimate N we need a suitable estimator of $\frac{1}{\mu}$.

If $X_1, X_2, .., X_m$ is a SRS from the population of interest, then the usual estimator of μ is the average of the sample, $\bar{X}$. Since, τ_X is known, a suggested estimator of N is

$$\hat{N}_{1SRS} = \frac{\tau_X}{\overline{X}_{1SRS}}.$$

$$E\left(\hat{N}_{1SRS}\right) = \tau_X \times E\left(\frac{1}{\overline{X}_{1SRS}}\right), \; Var\left(\hat{N}_{1SRS}\right) = \tau_X^2 \times \text{var}\left(\frac{1}{\overline{X}_{1SRS}}\right).$$

The mean squared error (MSE) of this estimator is:

$$MSE\left(\hat{N}_{1SRS}\right) = \tau_X^2 \; \text{var}\left(\frac{1}{\overline{X}_{1SRS}}\right) + \left(\tau_X \times E\left(\frac{1}{\overline{X}_{1SRS}}\right) - N\right)^2.$$

Note that if $\overline{X}$ is always positive, then since $\left(x^{-1}\right)$ a convex function, by **Jessen's Inequality** we have

$E\left(\hat{N}_{1SRS}\right) = \tau_X \times E\left(\frac{1}{\overline{X}_{1SRS}}\right) \geq \tau_X \times \left(\frac{1}{\mu}\right) = N$. Thus, $\hat{N}_{1SRS}$ is a positively biased estimator of N. While, if $\overline{X}$ is always negative, $E\left(\hat{N}_{1SRS}\right) \leq N$, it is negatively biased.

<u>Estimation the Population Size Using RSS</u>

Let $X_{(1:m)}, ..., X_{(m:m)}$ be a RSS of size m. Let $\overline{X}_{1RSS} = \sum_{i=1}^{m} X_{(i:m)} / m$ it is an unbiased estimator of μ (i.e. $E\left(\overline{X}_{1RSS}\right) = \mu$). Thus, a suitable RSS estimator for the population size is

$$\hat{N}_{1RSS} = \frac{\tau_X}{\overline{X}_{1RSS}}.$$

$$MSE\left(\hat{N}_{1RSS}\right) = \tau_X^2 \; \text{var}\left(\frac{1}{\overline{X}_{1RSS}}\right) + \left(\tau_X E\left(\frac{1}{\overline{X}_{1RSS}}\right) - N\right)^2.$$

The efficiency of $\hat{N}_{1RSS}$ w.r.t. $\hat{N}_{1SRS}$ is

$$Eff\left(\hat{N}_{1RSS}; \hat{N}_{1SRS}\right) = MSE\left(\overline{X}_{1SRS}\right) / MSE\left(\overline{X}_{1RSS}\right).$$

It is not possible to find the MSE theoretically, so we find it based on simulation from some well-known distributions: Gamma, Beta and Uniform. Tables (4.1), (4.2) & (4.3) contain the Bias, MSE, of each of the two estimators and the efficiency of $\hat{N}_{1RSS}$ w.r.t. $\hat{N}_{1SRS}$ for different values of N and m.

Table (4.1): Bias, MSE of $\hat{N}_{1RSS}$ & $\hat{N}_{1SRS}$ & Efficiency for Gamma (2,1)

N	m	$Bias\left(\hat{N}_{1SRS}\right)$	$MSE\left(\hat{N}_{1SRS}\right)$	$Bias\left(\hat{N}_{1RSS}\right)$	$MSE\left(\hat{N}_{1RSS}\right)$	$Eff\left(\hat{N}_{1RSS}, \hat{N}_{1SRS}\right)$
10000	**2**	3159	96655381	2024	40917200	2.362
	3	2039	39322421	984	13942660	2.820

	4	1349	22833941	616	7519040	3.037
	5	1167	17210250	420	4683529	3.675
5000	**2**	1686	25168221	1036	10852461	2.319
	3	1005	10644658	514	3581116	2.972
	4	732	6213451	298	1936500	3.208
	5	558	41520518	192	1152250	3.603
1000	**2**	336	956256.2	200	388807	2.459
	3	190	398662.42	96	135999	2.931
	4	133	221738.12	61	74016	2.995
	5	107	158533.85	39	44354	3.574

Table (4.2): Bias, MSE of $\hat{N}_{1RSS}$ & $\hat{N}_{1SRS}$ & Efficiency for Beta (4,3)

N	**m**	$Bias\left(\hat{N}_{1SRS}\right)$	$MSE\left(\hat{N}_{1SRS}\right)$	$Bias\left(\hat{N}_{1RSS}\right)$	$MSE\left(\hat{N}_{1RSS}\right)$	$Eff\left(\hat{N}_{1RSS},\hat{N}_{1SRS}\right)$
10000	**2**	572	8044468	317	4327625	1.859
	3	399	4738801	182	1921000	2.467
	4	299	3120482	117	1095289	2.849
	5	206	2389460	58	692264	3.451
5000	**2**	286	1895952	172	1051000	1.803
	3	160	1020343	81	452935	2.253
	4	125	707250	48	250417	2.824
	5	103	543676	30	166845	3.258
1000	**2**	59	79681	38	43839	1.817
	3	31	41551	15	18882	2.201
	4	23	29668	10	11028	2.690
	5	19	23008	6	6692	3.437

Table (4.3): Bias, MSE of $\hat{N}_{1RSS}$ & $\hat{N}_{1SRS}$ & Efficiency for Uniform (1,2)

N	**m**	$Bias\left(\hat{N}_{1SRS}\right)$	$MSE\left(\hat{N}_{1SRS}\right)$	$Bias\left(\hat{N}_{1RSS}\right)$	$MSE\left(\hat{N}_{1RSS}\right)$	$Eff\left(\hat{N}_{1RSS};\hat{N}_{1SRS}\right)$
10000	**2**	199	2130517	131	1351186	1.577
	3	127	1345538	60	642001	2.096
	4	83	996914	41	373781	2.667
	5	88	778628	24	243625	3.196
5000	**2**	98	544589	61	345995	1.574
	3	67	348679	35	164016	2.125

	4	51	254304	17	95897	2.651
	5	44	196770	13	62068	3.170
1000	2	17	20471	12	13459	1.521
	3	13	13353	7	6333	2.108
	4	10	9755	4	3822	2.552
	5	9	7792	3	2470	3.154

Based on the contents of the above tables, it can be seen that the Bias and MSE of each the two estimators, are decreasing in m and N. Both estimators are positively biased. The Bias and MSE of $\hat{N}_{1RSS}$ is less than that of $\hat{N}_{1SRS}$ based on the same m and N. $Eff\left(\hat{N}_{1RSS};\hat{N}_{1SRS}\right)$ is always larger than 1 and it is increasing in m for fixed N. N has a very slight effect on the efficiency.

5. Estimation the Population Size Using MERSS

MERSS is one of the main variations of RSS. It is obtained by measuring the judgment maximum, minimum or both of random samples of size $1,2,\dots,m$.

Let $X_{(1:1)},\dots,X_{(m:m)}$ be a MERSS (max.) of size m. Let $\overline{X}_{MERSSm}=\frac{\sum_{i=1}^{m}X_{(i:i)}}{m}$.

Since $\tau_X=N\mu\rightarrow N=\frac{\tau_X}{\mu}$, a suitable estimator for the population size is

$$\hat{N}_{MERSSm}=\frac{\tau_X}{\overline{X}_{MERSSm}},$$

with MSE given by

$$MSE\left(\hat{N}_{MERSSm}\right)=\tau_X^2\times\operatorname{var}\left(\frac{1}{\overline{X}_{MERSSm}}\right)+\left(\tau_X E\left(\frac{1}{\overline{X}_{MERSSm}}\right)-N\right)^2.$$

Tables (5.1), (5.2) and (5.3) contain the Bias, MSE, and the efficiency of $\hat{N}_{MERSS}$ w.r.t. $\hat{N}_{SRS}$ for different values of N and m.

Table (5.1): Bias, MSE of $\hat{N}_{MERSSm}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSSm};\hat{N}_{SRS}\right)$ for Gamma (2,1)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSSm}\right)$	$MSE\left(\hat{N}_{MERSSm}\right)$	$Eff\left(\hat{N}_{MERSSm},\hat{N}_{SRS}\right)$
10000	2	3344	89062961	292	29864113	2.982
	3	2011	38524505	-1624	10956456	3.516
	4	1412	24093145	-2582	10554719	2.282
	5	1128	16780228	-3174	12364568	1.357

5000	**2**	1702	22451400	173	7449119	3.013
	3	1050	10931562	-838	2754566	3.968
	4	709	6184321	-1304	2670125	2.316
	5	555	4187682	-1583	3067456	1.365
1000	**2**	334	1039436	23	275944	3.766
	3	202	415960	-163	109415	3.801
	4	148	242769	-260	105290	2.305
	5	107	164295	-315	121505	1.352

Table (5.2): Bias, MSE of $\hat{N}_{MERSSm}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSSm};\hat{N}_{SRS}\right)$ for Bata (4,3)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSSm}\right)$	$MSE\left(\hat{N}_{MERSSm}\right)$	$Eff\left(\hat{N}_{MERSSm},\hat{N}_{SRS}\right)$
10000	**2**	604	8289041	-488	4019807	2.062
	3	325	4386386	-1133	2836342	1.546
	4	259	3101645	-1502	3074425	1.008
	5	184	2323025	-1748	3564981	0.651
5000	**2**	273.2	1907413	-227	970680	1.965
	3	161	1005556	-541	649699	0.646
	4	128	706191	-728	723554	0.976
	5	95	555070	-852	846446	0.655
1000	**2**	52	77462	-50	40650	1.905
	3	37	44640	-111	27390	1.629
	4	26	30068	-150	30554	0.984
	5	20	23030	-176	36023	0.639

Table (5.3): Bias, MSE of $\hat{N}_{MERSSm}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSSm},\hat{N}_{SRS}\right)$ for Uniform (1,2)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSSm}\right)$	$MSE\left(\hat{N}_{MERSSm}\right)$	$Eff\left(\hat{N}_{MERSSm},\hat{N}_{SRS}\right)$
10000	**2**	197	2112409	-373.6	1519967	1.389
	3	137	1362050	-774.4	1254500	1.085
	4	95	951866	-1026	1425508	0.667
	5	79	775370	-1194	1663239	0.466
5000	**2**	105	538412	-186	390700	1.378
	3	72	351266	-390	322321	1.089

	4	52	254628	-516	364148	0.699
	5	40.8	201831	-609	433076	0.466
1000	**2**	19	20711	-40	15852	1.306
	3	12	13092	-79	13093	0.999
	4	10	10038	-103	14700	0.682
	5	7	7707	-122	17393	0.443

<u>Estimation of N using MERSS (Minimum)</u>

Let $X_{(1:1)}, X_{(1:2)}, \ldots, X_{(1:m)}$ be a MERSS (min.) of size m from the U(0, θ), then

$$\hat{N}_{MERSS1} = \frac{\tau_X}{\overline{X}_{MERSS1}}.$$

Tables (5.4), (5.5) and (5.6) contains the Bias, MSE, and the efficiency of $\hat{N}_{MERSS1}$ w.r.t $\hat{N}_{SRS}$ for different value of N size and m .

Table (5.4): The Bias, MSE of $\hat{N}_{MERSS1}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSS1};\hat{N}_{SRS}\right)$ for Gamma (2,1)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSS1}\right)$	$MSE\left(\hat{N}_{MERSS1}\right)$	$Eff\left(\hat{N}_{MERSS1},\hat{N}_{SRS}\right)$
10000	**2**	553	8292085	1635	12620941	0.657
	3	359	4627522	2195	11412649	0.405
	4	270	3124909	2732	12598580	0.248
	5	207	2356290	3272	14957828	0.157
5000	**2**	271	1857373	775	2841169	0.653
	3	170	1004908	1046	2577012	0.389
	4	125	694691	1290	2757603	0.252
	5	91	524165	1530	3266038	0.1604
1000	**2**	51	75771	159	117938	0.642
	3	33	43250	215	105804	0.408
	4	28	29475	269	117899	0.249
	5	21	23216	318	138641	0.167

Table (5.5): Bias, MSE of $\hat{N}_{MERSS1}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSS1},\hat{N}_{SRS}\right)$ for Beta (4,3)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSS1}\right)$	$MSE\left(\hat{N}_{MERSS1}\right)$	$Eff\left(\hat{N}_{MERSS1},\hat{N}_{SRS}\right)$
10000	**2**	3243	95636125	6442	163425128	0.585

	3	2024	40481600	6901	114749050	0.352
	4	1420	23928161	8181	121836861	0.196
	5	1149	16922701	9414	135518500	0.124
5000	2	1639	23634731	3393	88337674	0.267
	3	989	9897303	3574	30130458	0.328
	4	728	5983349	4227	31850544	0.1878
	5	567	4273178	4885	36729733	0.116
1000	2	337	1005447	634	1716189	0.585
	3	196	401022	707	1248294	0.321
	4	143	246711	811	1211298	0.203
	5	110	169088	942	1355510	0.124

Table (5.6): Bias, MSE of $\hat{N}_{MERSS1}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSS1},\hat{N}_{SRS}\right)$ for Uniform (1,2)

N	**m**	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSS1}\right)$	$MSE\left(\hat{N}_{MERSS1}\right)$	$Eff\left(\hat{N}_{MERSS1},\hat{N}_{SRS}\right)$
10000	2	180	2048800	760	2582656	0.793
	3	115	1331129	1112	2504420	0.531
	4	90	976356	1418	2924660	0.333
	5	71	768917	1674	3517992	0.218
5000	2	96	529865	386	667684	0.793
	3	59	332189	558	628218	0.528
	4	47	250550	716	745602	0.336
	5	34	195434	844	888313	0.220
1000	2	20	21190	79	27058	0.783
	3	14	13757	114	26497	0.519
	4	9	9801	145	30308	0.323
	5	6.8	7878	169	35566	0.221

Based on the contents of the above tables, we note that the values of the efficiency are conflicting; this may be due to the fact that the average is not unbiased. As in the case of SRS and RSS. For example, if $X_{(1:1)}, X_{(2:2)}, \ldots, X_{(m:m)}$ is a MERSS (max.) of size m from the U(0, θ), then $\frac{X_{(i:i)}}{\theta}$ is beta(i, 1), hence, $E\left(X_{(i:i)}\right) = \frac{i}{i+1}\theta$ & $Var(X_{(i:i)}) = \frac{i}{(i+1)^2(i+2)}\theta^2$. Similar comments can be said for MERSS (min.)

<u>Estimation of N using MERSS (Minimum and maximum (Both))</u>

In order to overcome the above problem, we take MERSS (max.) and MERSS (min.), (MERSS both). Tables (5.7, 5.8, 5.9) contain the Bias, MSE and the efficiency of $\hat{N}_{MERSS}$ w.r.t $\hat{N}_{SRS}$ for different values of the population size and set size m .

Table (5.7): Bias, MSE of $\hat{N}_{MERSS}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSS},\hat{N}_{SRS}\right)$ for Gamma (2,1)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSS}\right)$	$MSE\left(\hat{N}_{MERSS}\right)$	$Eff\left(\hat{N}_{MERSS},\hat{N}_{SRS}\right)$
10000	**4**	1430	24144301	1139	17805290	1.356
	6	942	124269731	503	7914833	1.570
5000	**4**	768.5	6461521	627	4679396	1.381
	6	442.2	3119983	268	2101648	1.484
1000	**4**	139.1	224286	116	171009	1.312
	6	81.8	119117	52	77931	1.528

Table (5.8): Bias, MSE of $\hat{N}_{MERSS}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSS},\hat{N}_{SRS}\right)$ for Beta (4,3)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSS}\right)$	$MSE\left(\hat{N}_{MERSS}\right)$	$Eff\left(\hat{N}_{MERSS},\hat{N}_{SRS}\right)$
10000	**4**	269	3184057	220	2525876	1.261
	6	150	1858525	135	1317825	1.410
5000	**4**	117.8	693182	80	546657	1.268
	6	79.9	434100	64	302270	1.436
1000	**4**	27.2	30489	20	23603	1.291
	6	17.4	18018	13	12858	1.401

Table (5.9): Bias, MSE of $\hat{N}_{MERSS}$ & $\hat{N}_{SRS}$ & $Eff\left(\hat{N}_{MERSS},\hat{N}_{SRS}\right)$ for Uniform (1,2)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MERSS}\right)$	$MSE\left(\hat{N}_{MERSS}\right)$	$Eff\left(\hat{N}_{MERSS},\hat{N}_{SRS}\right)$
10000	**4**	99	960426	96	813825	1.180
	6	51	633037	54	439837	1.439
5000	**4**	45	253819	38	201140	1.261
	6	33	161022	18	115313	1.396
1000	**4**	9	9816	10	8432	1.164
	6	8	6604	4	4547	1.452

Based on the contents of the above tables, it can be seen that the Bias and MSE of each of the two estimators, $\hat{N}_{SRS}$ and $\hat{N}_{MERSS}$, are decreasing in m and N. The efficiency is between 1 and 1.6 for all chosen distributions, the efficiency is increasing in m. $\hat{N}_{MERSS}$ is more efficient than $\hat{N}_{SRS}$.

6. Estimation of N using Median ranked set sampling (MRSS)

Median ranked set sampling is another variation of RSS(MRSS) introduced by Muttlak (1997). For simplicity, assume m is odd.

Median RSS (MRSS) can be obtained as follows:

(1) A SRS, $X_1, X_2, ..., X_m$ is taken from the population, the median of the sample, denoted by $V_1 = X_{\left(\frac{m+1}{2}:m\right)}$ is identified by judgment.

(2) Step 1 is repeated m times to obtain m medians.

The obtained sample, denoted by $V_1, V_2, ..., V_m$ is a called MRSS. Let $\overline{V}_{MRSS} = \frac{\sum_{i=1}^{m} V_i}{m}$. Again, a suitable estimator of N is

$$\hat{N}_{MRSS} = \frac{\tau_X}{\overline{V}_{MRSS}},$$

and the corresponding MSE is

$$MSE\left(\hat{N}_{MRSS}\right) = \tau_X^2 \times \text{var}\left(\frac{1}{\overline{V}_{MRSS}}\right) + \left(\tau_X E\left(\frac{1}{\overline{V}_{MRSS}}\right) - N\right)^2 .$$

Tables (6.1), (6.2) and (6.3) contain the Bias, MSE and the efficiency of $\hat{N}_{MRSS}$ w.r.t $\hat{N}_{SRS}$ for different values of N and m.

Table (6.1): Bias, MSE of $\hat{N}_{MRSS}$ & $\hat{N}_{SRS}$ & efficiency for Beta (4,3)

N	**m**	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MRSS}\right)$	$MSE\left(\hat{N}_{MRSS}\right)$	$Eff\left(\hat{N}_{MRSS}, \hat{N}_{SRS}\right)$
10000	**3**	366	4535560	87	1803169	2.52
	5	217	2318138	-35.9	663559.25	3.49
	7	146	1616485	-71.7	400027.45	4.04
5000	**3**	168	1022432.41	38.6	412114.6	2.48
	5	103	534061.25	-3.2	152188.25	3.51
	7	65.9	361349.06	-13.4	93632.05	3.86
1000	**3**	34.8	42990.4	10.9	18021.25	2.39
	5	21.2	22680.25	1.3	6627.65	3.42
	7	13.9	15843.22	-1.43	4083.6993	3.88

Table (6.2): Bias, MSE of $\hat{N}_{MRSS}$ & $\hat{N}_{SRS}$ & efficiency for Uniform (1,2)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MRSS}\right)$	$MSE\left(\hat{N}_{MRSS}\right)$	$Eff\left(\hat{N}_{MRSS},\hat{N}_{SRS}\right)$
10000	**3**	133	1335593	66	764740	1.75
	5	90	770229	26	312040	2.47
	7	53	548930	24	200385	2.74
5000	**3**	56	342675.29	44.3	201324.74	1.70
	5	34.8	198080.73	23.8	82878.05	2.39
	7	32.1	138374.77	23.7	54943.93	2.52
1000	**3**	13.3	13150.1	10.5	8354.89	1.57
	5	5.85	7707.4	5.4	3439.72	2.24
	7	5.4	5789.97	4.8	2241.45	2.58

Table (6.3): Bias, MSE of $\hat{N}_{MRSS}$ & $\hat{N}_{SRS}$ & efficiency values for G(2,1)

N	m	$Bias\left(\hat{N}_{SRS}\right)$	$MSE\left(\hat{N}_{SRS}\right)$	$Bias\left(\hat{N}_{MRSS}\right)$	$MSE\left(\hat{N}_{MRSS}\right)$	$Eff\left(\hat{N}_{MRSS},\hat{N}_{SRS}\right)$
10000	**3**	139	1378877	81	798661	1.73
	5	79	777125	32	331649	2.34
	7	49	561905	13	211769	2.65
5000	**3**	66.8	348679.13	44	202908.89	1.72
	5	44	196769.96	16.3	82979.45	2.37
	7	29.1	143882.05	13.5	53266.41	2.70
1000	**3**	13.2	13583.88	9	7825	1.74
	5	7.7	7593.53	6.8	3238.49	2.34
	7	4.4	5333.77	5.6	2092.52	2.55

Based on the contents of the above tables, it can be seen that efficiency is increasing in m. $\hat{N}_{MRSS}$ is more efficient than $\hat{N}_{SRS}$. Clearly, the difference between the efficiencies for different values of N is negligible; so the efficiency does not really depend on N. In practice the easiest to use is MERSS, because we only need to arrange by judgment 2 observations.

7. Conclusions

In this paper, we considered the estimation of the population size using RSS, MERSS and MRSS. The suggested estimators are compared with their counter parts using SRS. It turned out that the suggested estimators are more efficient than the corresponding ones using SRS. In practice the easiest to use is MERSS, because we only need to arrange by judgment 2 observations. Simultaneous estimation of the population size and total using ranked sampling techniques based Capture recapture method is a suggested future work.

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