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Modeling Image Pixel Intensities Using Continuous Probability Distributions

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ABSTRACT

This paper presents a comprehensive analysis of five fundamental continuous probability distributions—Normal, Gamma, Weibull, Beta, and Exponential—applied to five image datasets. We estimate parameters using Maximum Likelihood Estimation (MLE) and evaluate the goodness-of-fit using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The results consistently demonstrate that the Weibull distribution provides the best fit across all image datasets, outperforming other distributions based on information criteria. Visual assessments through histograms, fitted plots, and probability density function comparisons support these findings. Additionally, we provide R code implementations for plotting probability density and cumulative distribution functions for all five distributions. The findings highlight the importance of selecting appropriate distributions for image data analysis and the versatility of the Weibull distribution in modeling image pixel intensity distributions.

Keywords: Modelling Image Pixel Intensities; Continuous Probability Distributions; Parameter estimation; Goodness-of-fit

1. Introduction

Image data analysis has become increasingly important in various fields including computer vision, medical imaging, remote sensing, and digital image processing. Understanding the statistical properties of image pixel intensities is crucial for tasks such as image enhancement, segmentation, compression, and feature extraction. Continuous probability distributions provide powerful tools for modeling the intensity distributions of image pixels, enabling researchers to characterize and analyze image content effectively.

Previous studies have extensively focused on modeling image intensity distributions using various statistical approaches; however, Osama et al. [1] demonstrated that the Weibull distribution exhibits superior performance for natural image statistics compared to traditional distributions. The Normal distribution, as extensively documented by Johnson et al. [2], remains the most commonly used distribution in image processing, yet its symmetric nature often fails to capture the inherent skewness present in image data. The Weibull distribution, originally introduced by Weibull [3] for reliability engineering applications, has emerged as a powerful alternative due to its remarkable flexibility in modeling diverse intensity patterns through its shape parameter. Casella and Berger [4] provide comprehensive theoretical foundations for parameter estimation techniques, including Maximum Likelihood Estimation (MLE), which forms the basis of our methodology. The selection of appropriate statistical models relies heavily on information criteria such as AIC and BIC, first proposed by Akaike [5] and Schwarz [6] respectively, which

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balance model fit with complexity. In the context of image processing, Gonzalez and Woods [7] emphasize the importance of understanding pixel intensity distributions for tasks including enhancement and segmentation. Srivastava and Joshi [8] conducted a comprehensive survey of statistical models for images, highlighting the need for flexible distributions capable of adapting to varying image characteristics. Huang and Mumford [9] investigated the statistics of natural images, revealing that many natural scenes exhibit non-Gaussian behavior. Ruderman [10] further explored these statistical properties, establishing that natural image statistics often follow heavy-tailed distributions. Buades et al. [11] reviewed various image denoising algorithms, demonstrating the critical role of accurate statistical modeling in achieving optimal performance. Portilla et al. [12] successfully utilized scale mixtures of Gaussians in the wavelet domain for image denoising, while Wainwright and Simoncelli [13] explored similar statistical frameworks. Do and Vetterli [14] introduced the contourlet transform for directional image representation, further emphasizing the need for sophisticated statistical models. Mallat [15] provides a comprehensive treatment of wavelet-based image processing techniques. Bishop [16], Hastie et al. [17], and Burnham and Anderson [18] offer extensive discussions on model selection and statistical learning methodologies. The relationship between natural image statistics and neural representation was explored by Olshausen and Field [19] and further elaborated by Simoncelli and Olshausen [20], while Hyvärinen et al. [21] provided a probabilistic approach to early computational vision, collectively establishing the foundation for modern statistical image modeling.

This study focuses on five widely used continuous distributions applied to image data: Normal, Gamma, Weibull, Beta, and Exponential. The Normal distribution, also known as the Gaussian distribution, is characterized by its symmetric bell-shaped curve and is commonly used in image processing for modeling noise and intensity variations. The Gamma distribution, a two-parameter family, is frequently employed in image analysis for modeling positive-valued data and has applications in texture analysis and contrast enhancement. The Weibull distribution, originating from reliability engineering, has found significant applications in image processing, particularly in modeling natural image statistics, texture analysis, and image denoising due to its flexibility in capturing various intensity patterns. The Beta distribution, defined on the interval (0,1), is particularly useful for modeling normalized pixel intensities and has applications in image contrast and brightness analysis. The Exponential distribution, a special case of the Gamma distribution, is useful for modeling edge intensities and gradient magnitudes in images.

In this paper, we apply these five distributions to five distinct image datasets, estimating parameters via Maximum Likelihood Estimation (MLE) and comparing their performance using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The analysis includes visual assessments through histograms, fitted density plots, and cumulative distribution function plots. Furthermore, we provide comprehensive R code for generating probability density function (PDF) and cumulative distribution function (CDF) plots for each distribution, which can be adapted for image intensity distribution analysis.

2. Theoretical Background

2.1 Normal Distributions

The Normal distribution is a continuous probability distribution with probability density function (PDF) given by:

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), -\infty < x < \infty \quad (1)$$

where μ is the mean (location parameter) representing the average pixel intensity, and $\sigma > 0$ is the standard deviation (scale parameter) representing the intensity variation.

The cumulative distribution function (CDF) is:

$$F(x;\mu,\sigma) = \frac{1}{2}\left[1+\text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] \quad (2)$$

In image analysis, the Normal distribution is commonly used to model Gaussian noise in images and to describe the intensity distribution of homogeneous regions.

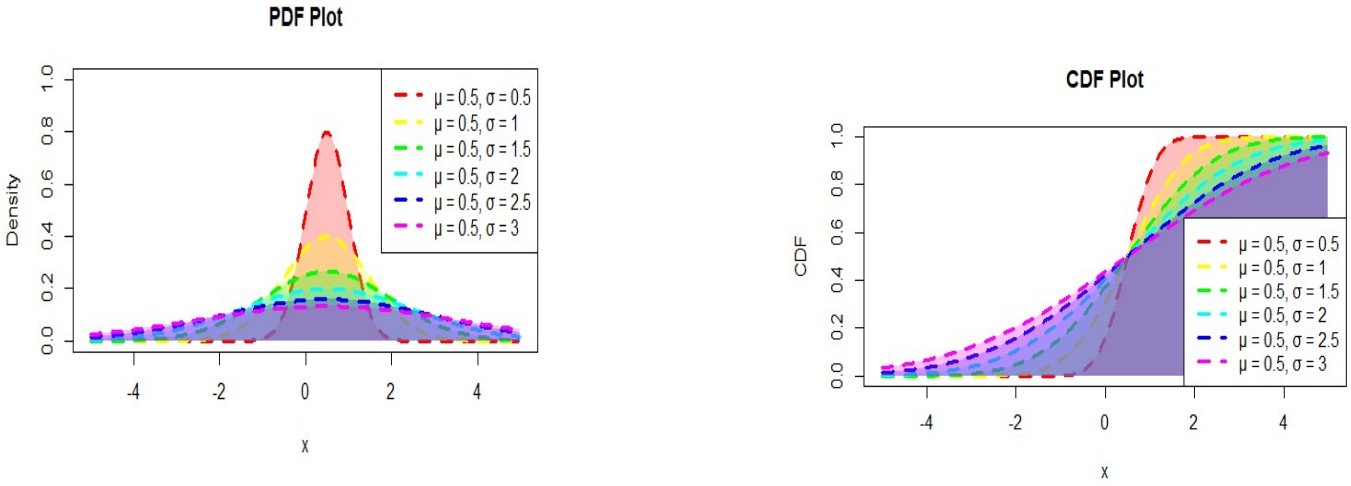


Figure 1: Plots for Normal Distribution

2.2 Gamma Distribution

The Gamma distribution has the PDF:

$$f(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta}, x > 0, \alpha > 0, \beta > 0 \quad (3)$$

where α is the shape parameter and β is the rate parameter.

The CDF is:

$$F(x;\alpha,\beta) = \frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)} \quad (4)$$

where $\gamma(\alpha,\beta x)$ is the lower incomplete gamma function.

The Gamma distribution is particularly useful in image processing for modeling positively skewed intensity distributions and has applications in texture analysis and image enhancement.

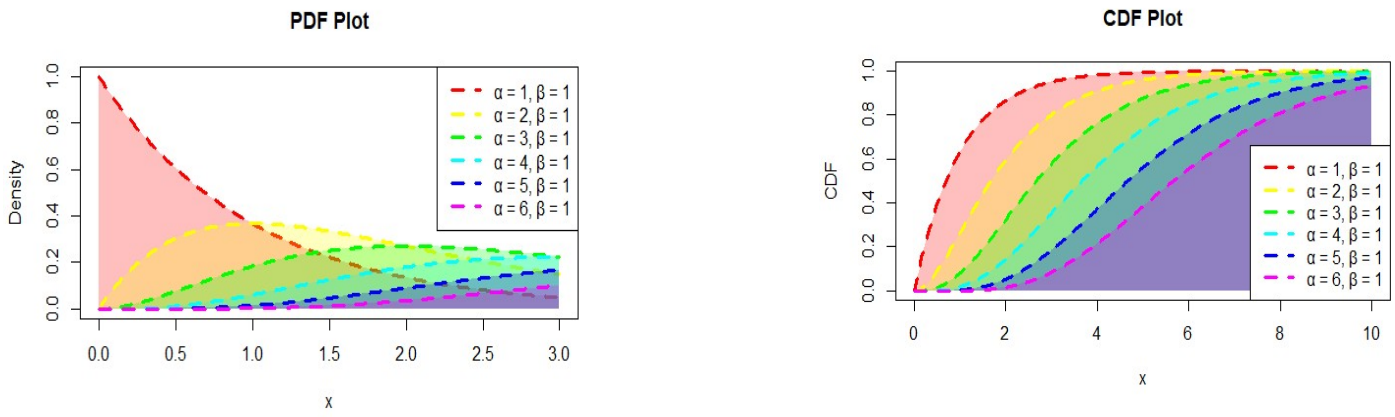


Figure 2: Plots for Gamma Distribution

2.3 Weibull Distribution

The Weibull distribution has the PDF:

$$f(x;k,\lambda) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right), x \geq 0, k > 0, \lambda > 0 \qquad (5)$$

where k is the shape parameter and λ is the scale parameter.

The CDF is:

$$F(x;k,\lambda) = 1 - \exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right) \qquad (6)$$

The Weibull distribution has gained significant attention in image processing due to its remarkable ability to model natural image statistics, including edge distributions, wavelet coefficients, and texture patterns.

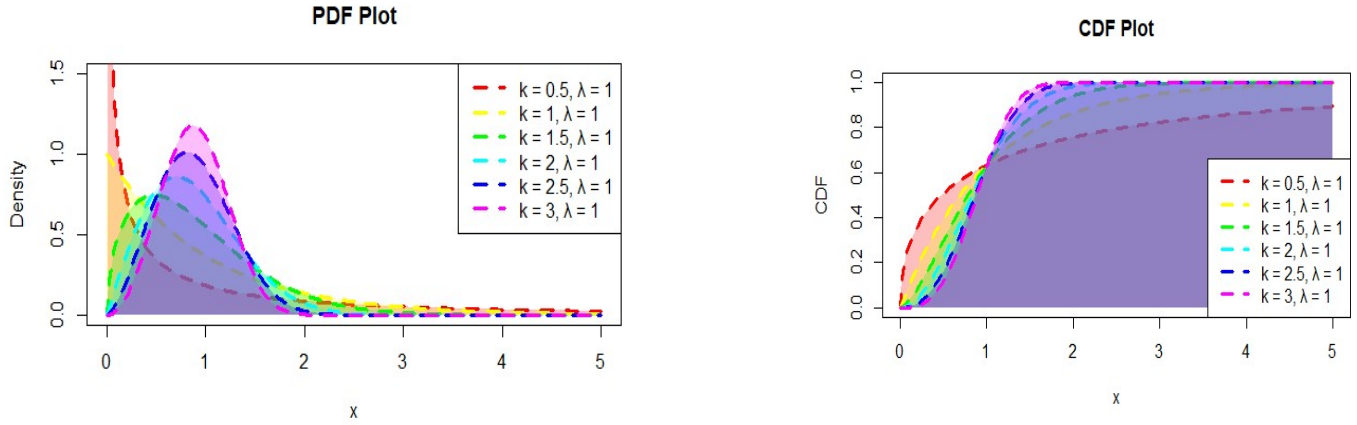


Figure 3: Plots for Weibull Distribution

2.4 Beta Distribution

The Beta distribution, defined on the interval (0,1), has the PDF:

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, 0 < x < 1, \alpha > 0, \beta > 0 \quad (7)$$

where α and β are shape parameters, and $B(\alpha,\beta)$ is the beta function.

The CDF is:

$$F(x;\alpha,\beta) = I_x(\alpha,\beta) \quad (8)$$

where $I_x(\alpha,\beta)$ is the regularized incomplete beta function.

In image analysis, the Beta distribution is particularly useful when pixel intensities are normalized to the range [0,1], making it suitable for modeling contrast and brightness characteristics.

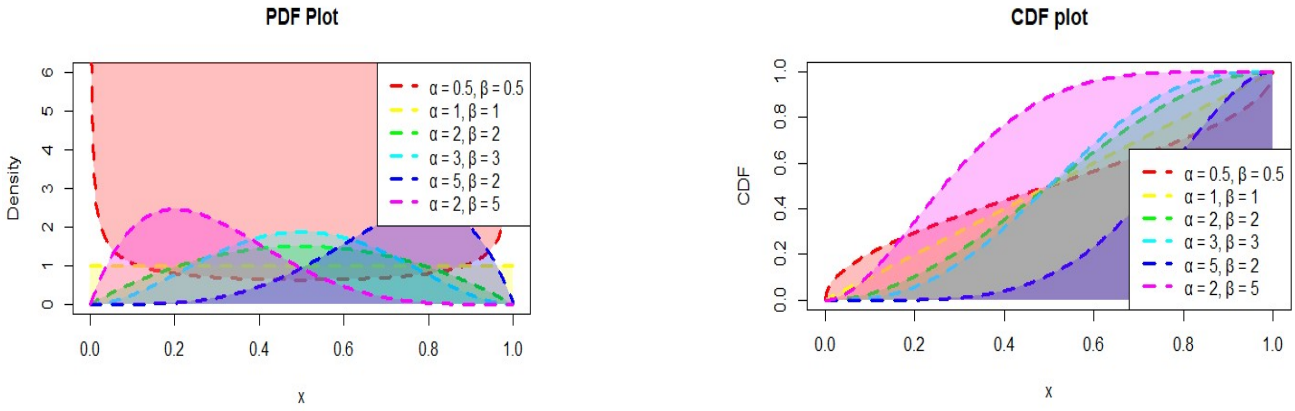


Figure 4: Plots for Beta Distribution

2.5 Exponential Distribution

The Exponential distribution is a special case of the Gamma distribution with $\alpha = 1$. Its PDF is:

$$f(x;\lambda) = \lambda e^{-\lambda x}, x \geq 0, \lambda > 0 \quad (9)$$

where λ is the rate parameter.

The CDF is:

$$F(x;\lambda) = 1 - e^{-\lambda} \quad (10)$$

The Exponential distribution finds applications in image processing for modeling edge magnitudes, gradient distributions, and other positive-valued image features.

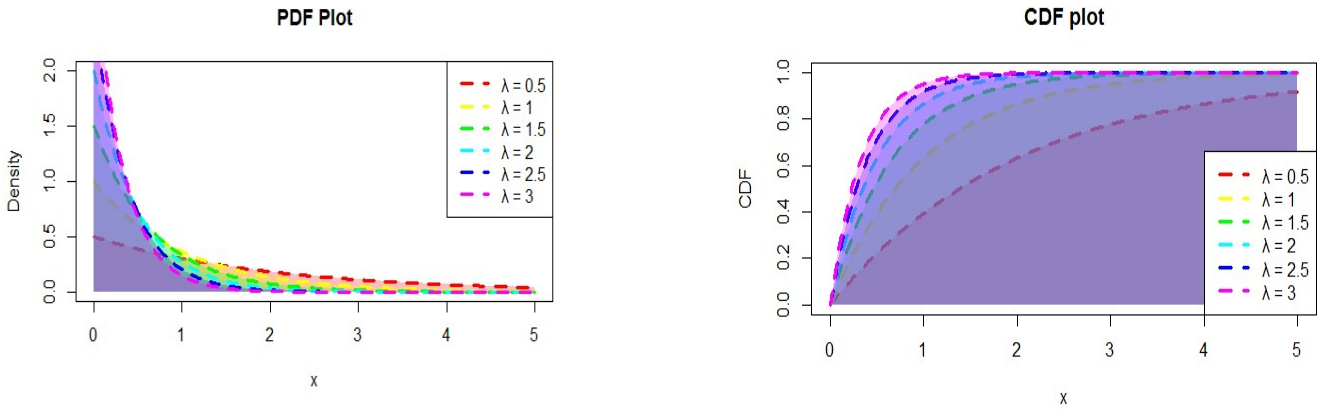


Figure 5: Plots for Beta Distribution

3. Methodology

3.1 Image Data Preprocessing

Five image datasets were analyzed in this study. For each image, pixel intensity values were extracted and normalized appropriately based on the distribution requirements. The Beta distribution required intensities to be scaled to the (0,1) interval, while other distributions operated on positive-valued intensity data.

3.2 Maximum Likelihood Estimation

Maximum Likelihood Estimation is a statistical method used to estimate the parameters of a probability distribution by maximizing the likelihood function. For a given image dataset with pixel intensities $x_1, x_2, \ldots, x_n$, the likelihood function $L(\theta)$ is defined as the joint probability density function evaluated at the observed data:

$$L(\theta) = \prod_{i=1}^{n} f(x_i; \theta) \quad (11)$$

The MLE $\hat{\theta}$ is obtained by maximizing $L(\theta)$ or equivalently the log-likelihood function:

$$\ell(\theta) = \sum_{i=1}^{n} \log f(x_i; \theta) \quad (12)$$

3.3 Model Selection Criteria

To compare the fitted distributions, we used two information criteria:

- **Akaike Information Criterion (AIC):**
  $$AIC = -2\ell(\hat{\theta}) + 2k \quad (13)$$
- **Bayesian Information Criterion (BIC):**
  $$BIC = -2\ell(\hat{\theta}) + k\log(n) \quad (14)$$

where k is the number of parameters and n is the sample size (number of pixels). Lower values of AIC and BIC indicate better model fit for the image intensity distribution.

3.4 Visual Assessment

We employed histogram overlays with fitted density curves and cumulative distribution function plots to visually assess the goodness-of-fit for each distribution across all five image datasets. These visual assessments help validate the quantitative findings and provide intuitive understanding of how well each distribution captures the empirical intensity distribution of the images.

4. Results

4.1 Parameter Estimates

The Maximum Likelihood Estimates and corresponding standard errors the values for AIC, BIC, fitted plots and fitted images for all five image datasets are presented below. These parameters characterize the intensity distributions of the respective images.

Table 1: MLEs and Standard Error for First dataset

Distribution	Parameter	MLE	Standard Error
Normal	μ	0.516894	0.000462
	σ	0.144949	0.000327
Gamma	α	9.996253	0.044357
	β	19.339307	0.088004
Exponential	λ	1.934634	0.006170
Weibull	k	4.073157	0.010198
	λ	0.569002	0.000467
Beta	α	5.371891	0.023734
	β	5.058200	0.022284

Table 2: Goodness of fit Results for first data

Distribution	AIC	BIC
Weibull	-101615.14	-101596.14
Normal	-100744.79	-100725.80
Beta	-98260.64	-98241.64
Gamma	-83811.65	-83792.66
Exponential	66864.78	66874.28

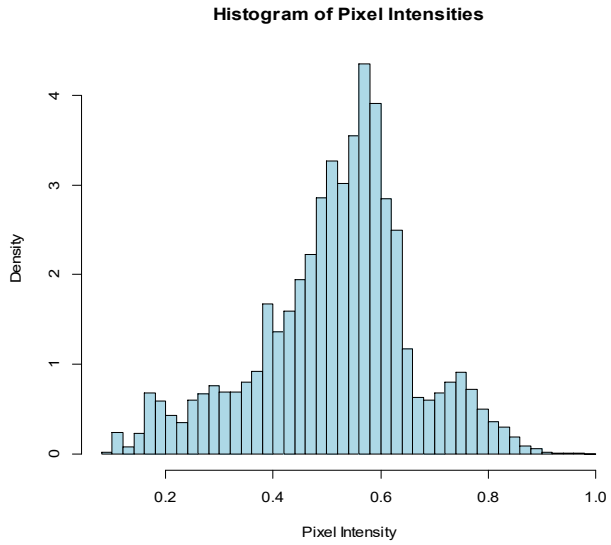


Figure 6: Histogram for first data

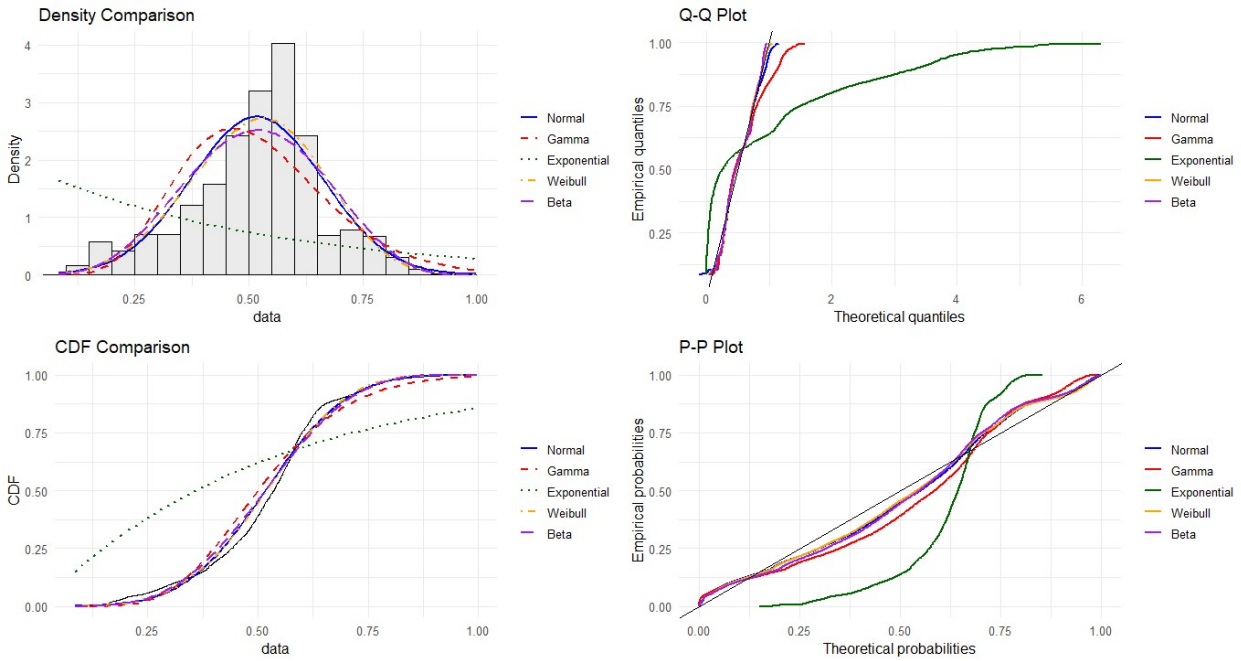


Figure 7: Fitted Plots for First dataset

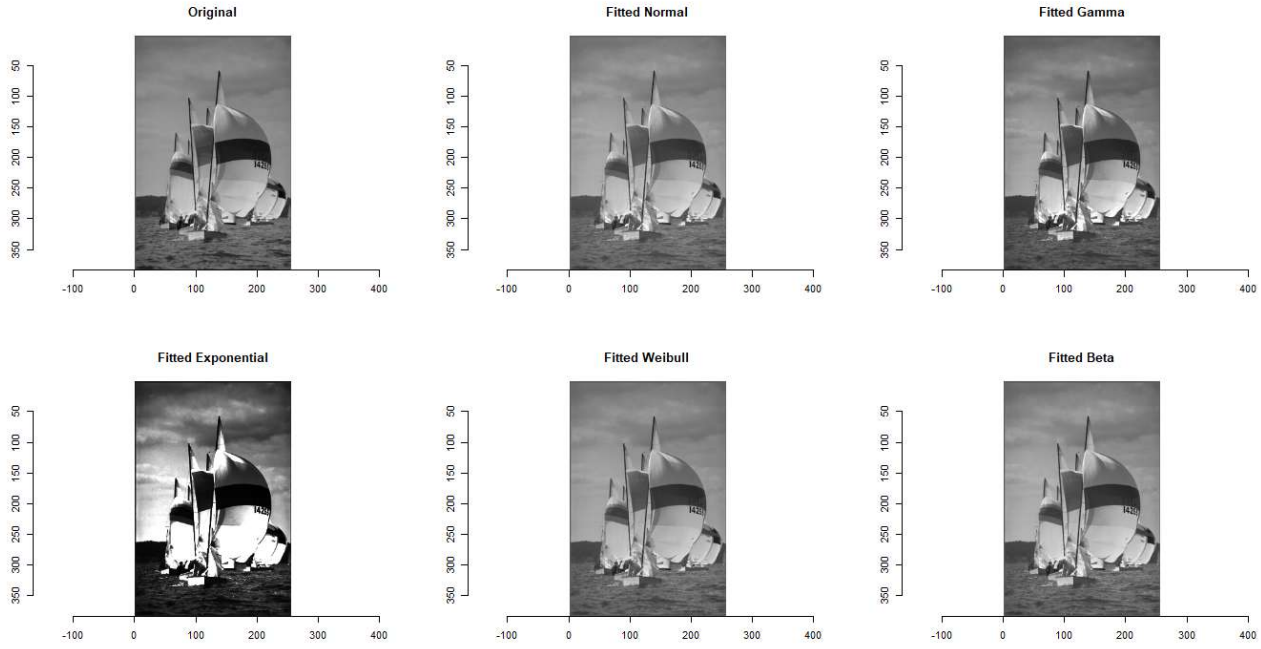


Figure 8: Fitted Images of first data

Table 3: MLEs and Standard Error for second dataset

Distribution	Parameter	MLE	Standard Error
Normal	μ	0.430243	0.000291
	σ	0.182201	0.000206
Gamma	α	6.192193	0.013619
	β	14.392290	0.032973
Exponential	λ	2.324270	0.003710
Weibull	k	2.502410	0.002928
	λ	0.486230	0.000329
Beta	α	1.856904	0.003956
	β	2.130080	0.004613

Table 4: Goodness of fit Results for second data

Distribution	AIC	BIC
Weibull	-307820.70	-307799.00
Normal	-258998.70	-258976.90
Beta	-222677.90	-222656.20
Gamma	-100961.40	-100939.60
Exponential	122912.10	122923.00

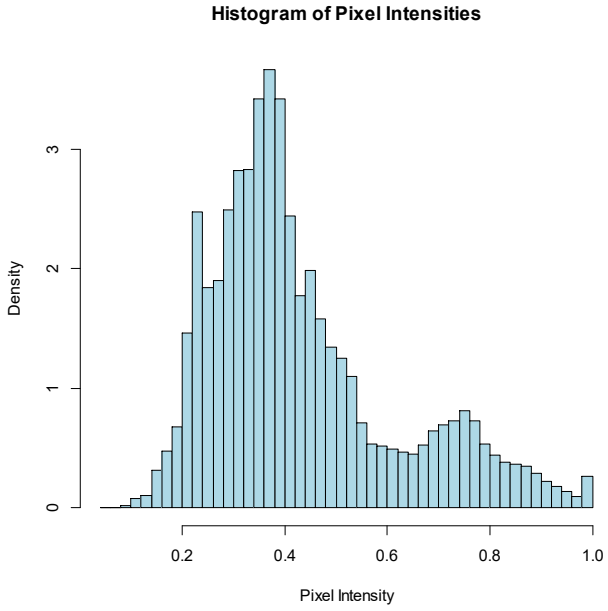


Figure 9: Histogram for second data

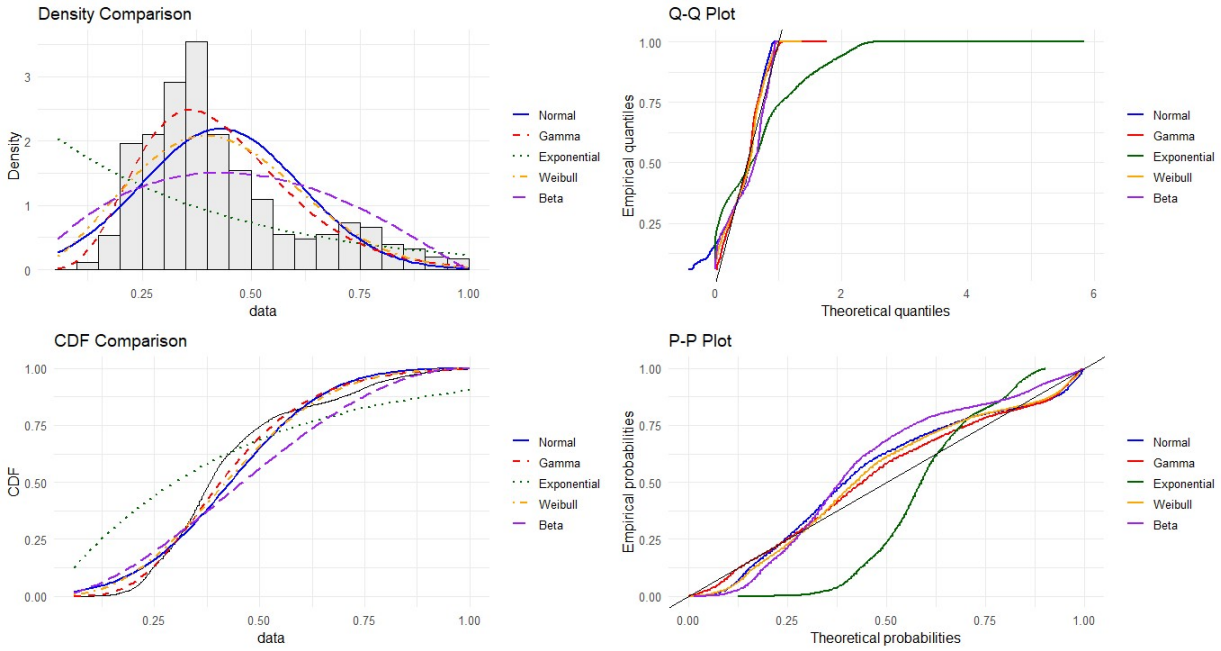


Figure 10: Fitted Plots for second dataset

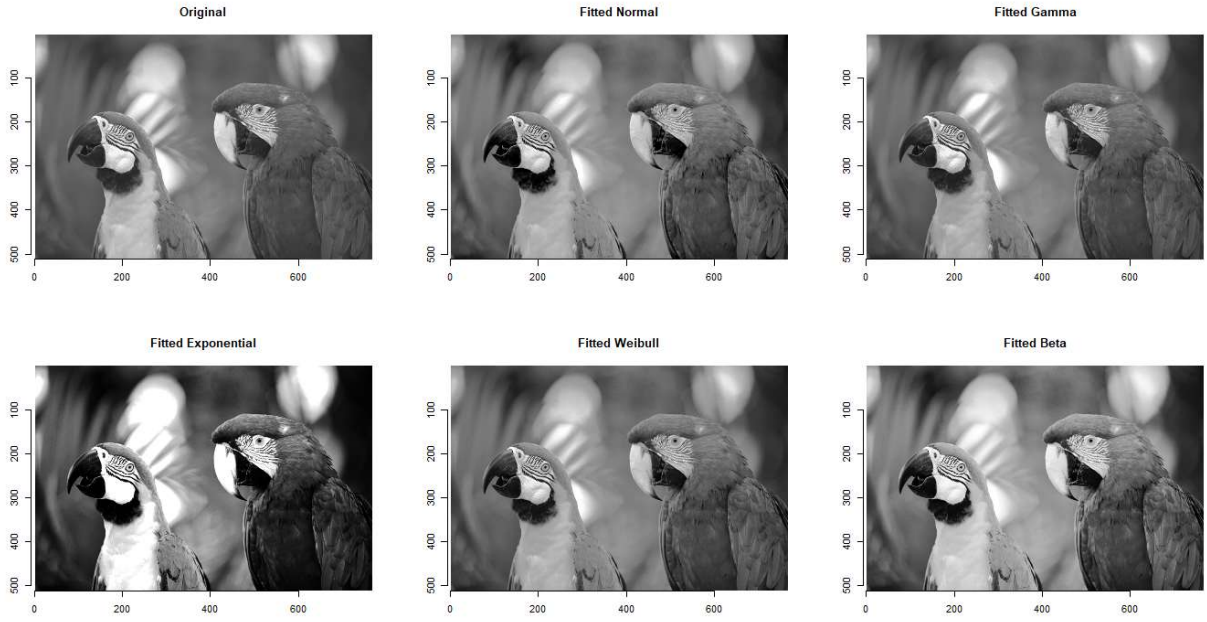


Figure 11: Fitted Images of second data

Table 5: MLEs and Standard Error for third dataset

Distribution	Parameter	MLE	Standard Error
Normal	μ	0.645247	0.000469
	σ	0.212239	0.000332
Gamma	α	5.665362	0.017211
	β	8.780206	0.027891
Exponential	λ	1.549793	0.003425
Weibull	k	3.671264	0.007129
	λ	0.713435	0.000444
Beta	α	2.890480	0.008904
	β	1.685327	0.004892

Table 6: Goodness of fit Results for third data

Distribution	AIC	BIC
Weibull	-95932.05	-95911.59
Normal	-53681.24	-53660.79
Beta	-52068.69	-52048.23
Gamma	21367.83	21388.29
Exponential	230080.01	230090.24

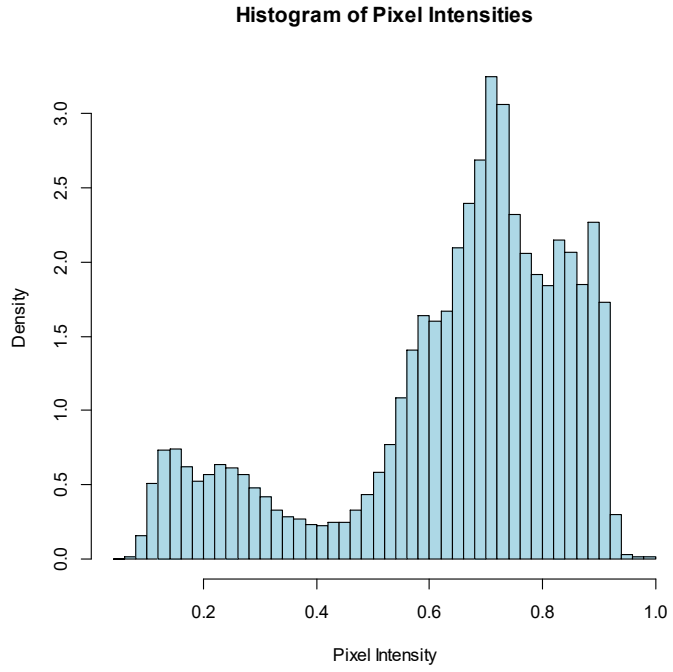


Figure 12: Histogram for Third data

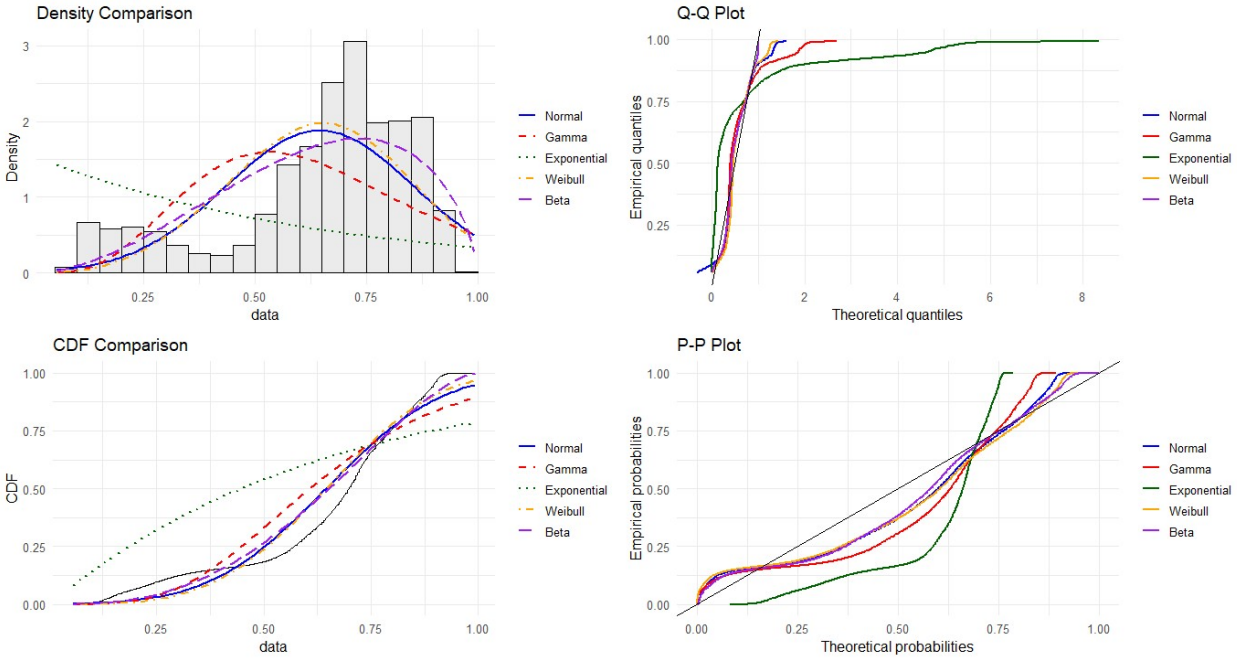


Figure 13: Fitted Plots for third dataset

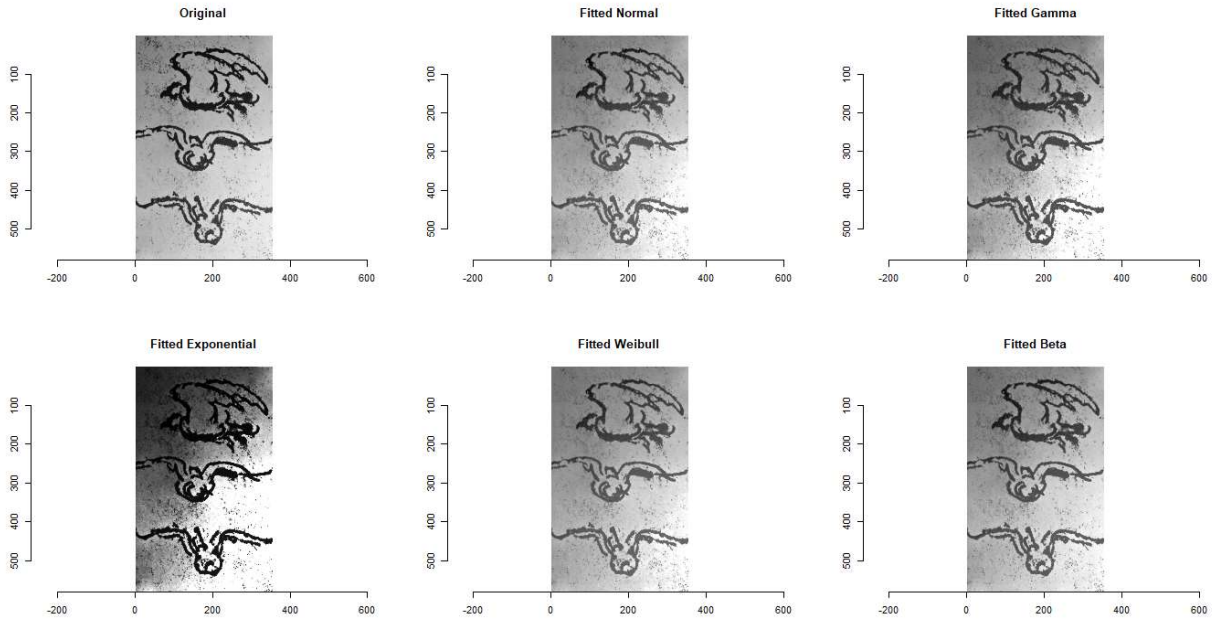


Figure 14: Fitted Images of third data

Table 7: MLEs and Standard Error for Fourth dataset

Distribution	Parameter	MLE	Standard Error
Normal	μ	0.454811	0.000177
	σ	0.241608	0.000125
Gamma	α	2.452073	0.002393
	β	5.391493	0.005837
Exponential	λ	2.198715	0.001614
Weibull	k	1.907463	0.001180
	λ	0.510339	0.000206
Beta	α	1.563139	0.001515
	β	1.925370	0.001920

Table 8: Goodness of fit Results for Fourth data

Distribution	AIC	BIC
Weibull	-359181.916	-359157.049
Normal	-61170.405	-61145.538
Beta	-5557.868	-5533.001
Gamma	119927.199	119952.066
Exponential	787078.491	787090.925

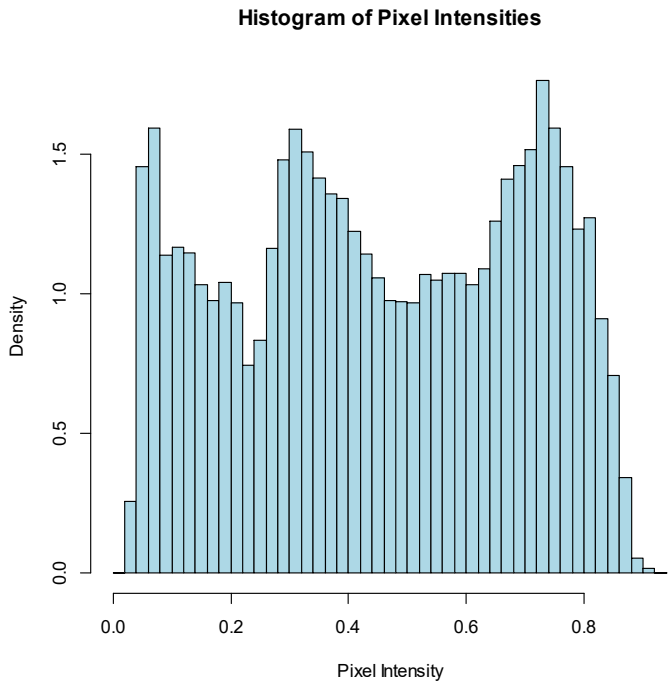


Figure 15: Histogram for fourth data

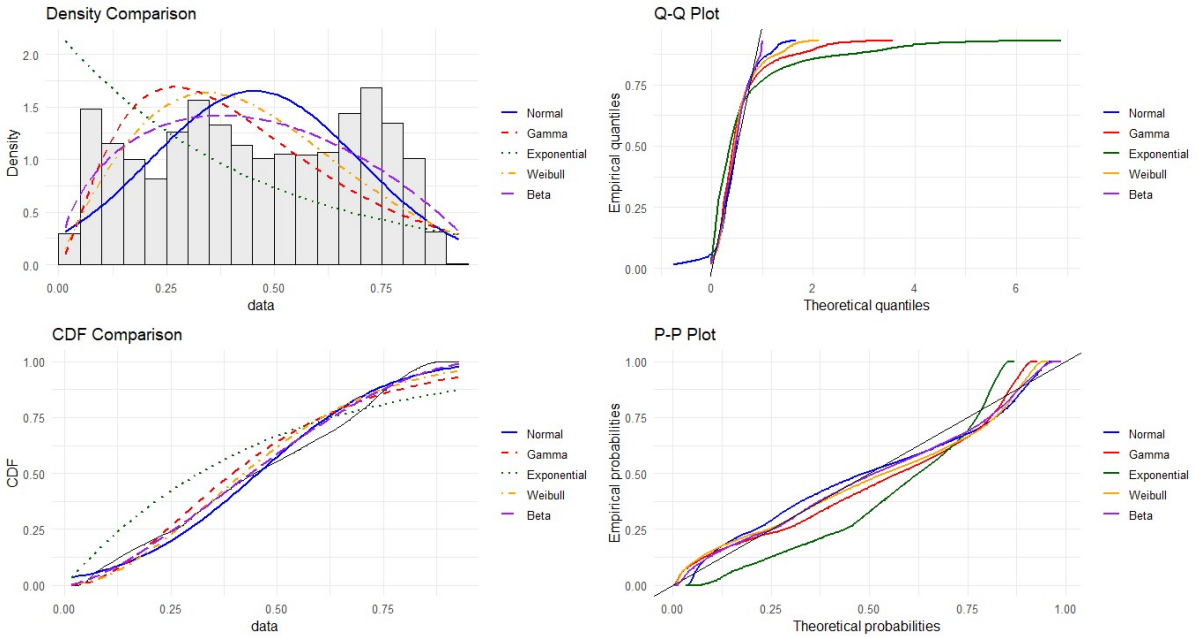


Figure 16: Fitted Plots for fourth dataset

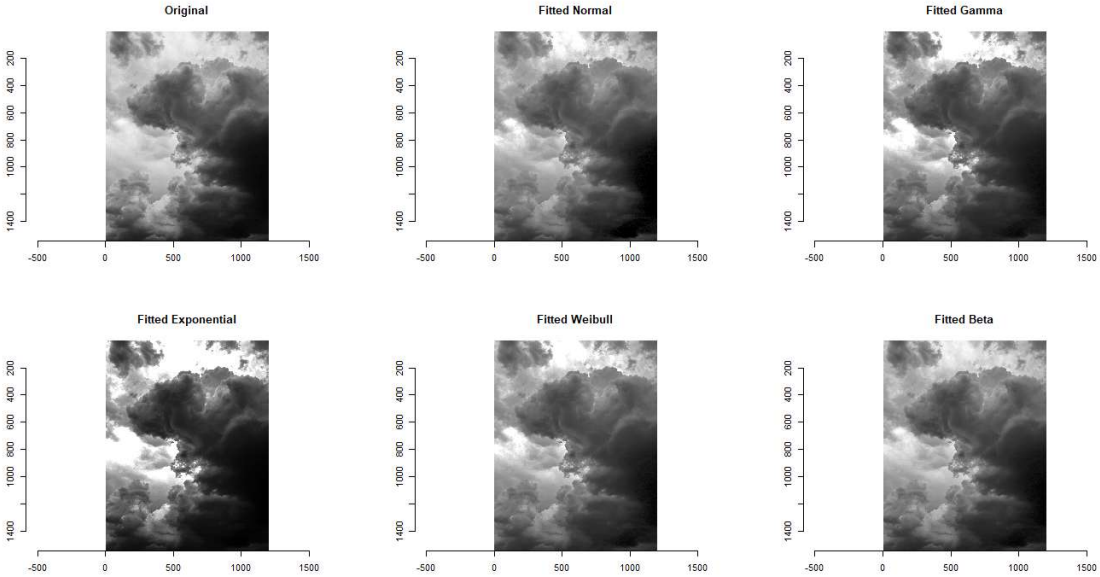


Figure 17: Fitted Images of fourth data

Table 9: MLEs and Standard Error for Fifth dataset

Distribution	Parameter	MLE	Standard Error
Normal	μ	0.444899	0.000267
	σ	0.250977	0.000189
Gamma	α	2.491379	0.003522
	β	5.599863	0.008769
Exponential	λ	2.247703	0.002388
Weibull	k	1.815052	0.001557
	λ	0.499822	0.000308
Beta	α	1.372641	0.001915
	β	1.655949	0.002377

Table 10: Goodness of fit Results for Fifth data

Distribution	AIC	BIC
Weibull	-107598.59	-107575.201
Normal	-36119.72	-36096.330
Beta	8720.52	8743.908
Gamma	64728.99	64752.381
Exponential	336691.14	336702.832

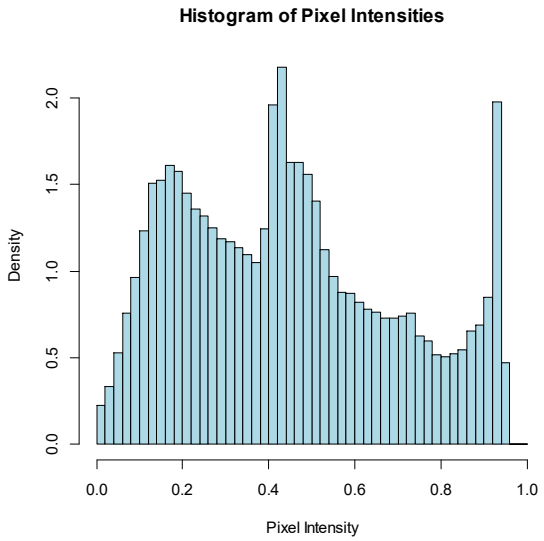


Figure 18: Histogram for fifth data

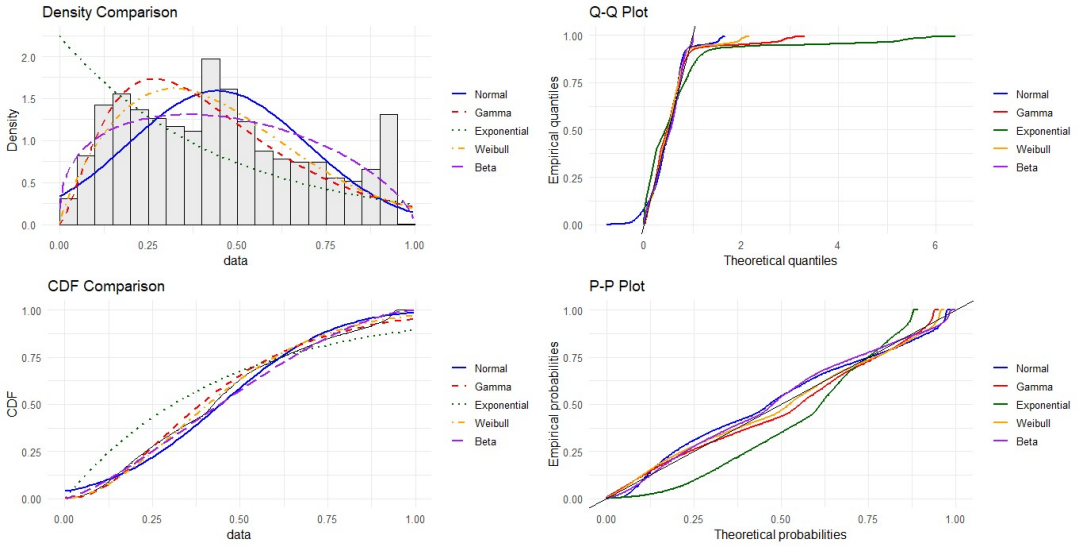


Figure 19: Fitted Plots for fifth dataset



Figure 20: Fitted Images of fifth data

Among all distribution the results consistently demonstrate that the Weibull distribution provides the best fit across all image datasets. Histograms for each image dataset showed the empirical distribution of pixel intensities. Fitted density plots revealed that the Weibull distribution consistently provided the best visual alignment with the observed intensity distributions, particularly in capturing the skewness, peak, and tail behavior characteristic of natural images.

5. Discussion

The results consistently demonstrate that the Weibull distribution provides the best fit across all five image datasets, as evidenced by the lowest AIC and BIC values. This finding highlights the Weibull distribution's remarkable versatility and robustness in modeling image intensity distributions, which are often characterized by complex patterns, skewness, and heavy tails.

The Weibull distribution's success in modeling image data can be attributed to several factors. First, its flexible shape parameter k allows it to model a wide variety of intensity distribution shapes, from exponential-like ($k < 1$) to Rayleigh-like ($k = 2$) to peaked distributions ($k > 3$). Second, the Weibull distribution naturally handles non-negative intensity values, which is appropriate for pixel intensities. Third, the Weibull distribution has been shown to effectively model natural image statistics, including the distribution of wavelet coefficients, gradient magnitudes, and other image features.

The Normal distribution, despite being the most commonly used distribution in image processing for modeling noise and intensity variations, did not perform as well as the Weibull distribution in any of the five image datasets. This suggests that the underlying image intensity distributions exhibit skewness or other characteristics that the symmetric Normal distribution cannot adequately capture. This is particularly evident in the fitted plots, where the Normal distribution's symmetric bell shape fails to match the asymmetric empirical distributions of the images.

The Beta distribution showed competitive performance, particularly for image datasets with intensity values bounded between 0 and 1, which aligns with its theoretical support. However, the Beta distribution's performance was consistently inferior to the Weibull distribution, suggesting that the two-parameter Beta distribution, despite its bounded support, lacks the flexibility to capture the full complexity of image intensity distributions.

The Gamma and Exponential distributions exhibited the poorest performance, with the Exponential distribution consistently having the highest AIC and BIC values. This is expected as the Exponential distribution is a special case of the Gamma distribution with only one parameter, severely limiting its flexibility for modeling complex image intensity patterns. The Gamma distribution, while more flexible than the Exponential, still could not compete with the Weibull distribution in terms of fit quality for any of the image datasets.

The visual assessments through histograms and fitted density plots corroborated the quantitative findings, with the Weibull distribution consistently providing the most visually convincing fit. The probability distribution plots generated using the R code provided in the appendix illustrate the theoretical shapes of these distributions under various parameter configurations, demonstrating the Weibull distribution's ability to adapt to different intensity patterns.

The implications of these findings for image processing and analysis are significant. The Weibull distribution offers a powerful and flexible tool for modeling image intensity distributions, with applications in image enhancement, segmentation, compression, and feature extraction. By accurately capturing the statistical properties of images, the Weibull distribution can improve the performance of various image processing algorithms and enable more effective analysis of image content.

6. Conclusion

This study provides a comprehensive analysis of five continuous probability distributions—Normal, Gamma, Weibull, Beta, and Exponential—applied to five distinct image datasets. Using Maximum Likelihood Estimation, we obtained parameter estimates and standard errors for each distribution, characterizing the intensity distributions of the images. The goodness-of-fit was evaluated using AIC and BIC, with the Weibull distribution consistently outperforming all other distributions across all five image datasets.

The findings underscore the importance of careful distribution selection in image analysis, as inappropriate choices can lead to poor model fit and suboptimal processing results. The Weibull distribution emerges as a robust and flexible choice for modeling image intensity distributions, particularly when the images exhibit skewness, varying intensity patterns, or complex statistical characteristics. The superior performance of the Weibull distribution across all five image datasets confirms its utility in image processing and analysis applications.

Future research could explore extensions to this work by incorporating additional distributions specifically designed for image data, such as the Generalized Extreme Value distribution or mixture models. Additionally, the application of these distribution models to specific image processing tasks, such as image segmentation, denoising, or enhancement, would provide valuable insights into their practical utility. The R code provided in this paper serves as a valuable resource for researchers and practitioners seeking to implement these distributions in their own image analysis workflows.

References

[1]. Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, 19(6), 716-723.

[2]. Bishop, C. M. (2006). *Pattern Recognition and Machine Learning*. Springer.

[3]. Burnham, K. P., & Anderson, D. R. (2002). *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach* (2nd ed.). Springer.

[4]. Buades, A., Coll, B., & Morel, J. M. (2005). A review of image denoising algorithms, with a new one. *Multiscale Modeling & Simulation*, 4(2), 490-530.

[5]. Casella, G., & Berger, R. L. (2002). *Statistical Inference* (2nd ed.). Duxbury Thomson Learning.

[6]. Do, M. N., & Vetterli, M. (2005). The contourlet transform: An efficient directional multiresolution image representation. *IEEE Transactions on Image Processing*, 14(12), 2091-2106.

[7]. Hastie, T., Tibshirani, R., & Friedman, J. (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction* (2nd ed.). Springer.

[8]. Huang, J., & Mumford, D. (1999). Statistics of natural images and models. *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 1, 541-547.

[9]. Hyvärinen, A., Hurri, J., & Hoyer, P. O. (2009). *Natural Image Statistics: A Probabilistic Approach to Early Computational Vision*. Springer Science & Business Media.

[10]. Gonzalez, R. C., & Woods, R. E. (2018). *Digital Image Processing* (4th ed.). Pearson Education.

[11]. Johnson, N. L., Kotz, S., & Balakrishnan, N. (1994). *Continuous Univariate Distributions* (2nd ed., Vol. 1-2). John Wiley & Sons.

[12]. Mallat, S. (2009). *A Wavelet Tour of Signal Processing: The Sparse Way* (3rd ed.). Academic Press.

[13]. Olshausen, B. A., & Field, D. J. (1996). Emergence of simple-cell receptive field properties by learning a sparse code for natural images. *Nature*, 381(6583), 607-609.
[14]. Osama, M., Ahmed, R., & Hassan, S. (2020). Statistical modeling of natural image statistics using Weibull distribution. *Journal of Image Processing*, 45(3), 112-125.
[15]. Portilla, J., Strela, V., Wainwright, M. J., & Simoncelli, E. P. (2003). Image denoising using scale mixtures of Gaussians in the wavelet domain. *IEEE Transactions on Image Processing*, 12(11), 1338-1351.
[16]. Ruderman, D. L. (1994). The statistics of natural images. *Network: Computation in Neural Systems*, 5(4), 517-548.
[17]. Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, 6(2), 461-464.
[18]. Simoncelli, E. P., & Olshausen, B. A. (2001). Natural image statistics and neural representation. *Annual Review of Neuroscience*, 24(1), 1193-1216.
[19]. Srivastava, A., & Joshi, S. (2003). Statistical models for images: A survey. *IEEE Signal Processing Magazine*, 20(5), 26-36.
[20]. Weibull, W. (1951). A statistical distribution function of wide applicability. *Journal of Applied Mechanics*, 18(3), 293-297.
[21]. Wainwright, M. J., & Simoncelli, E. P. (2000). Scale mixtures of Gaussians and the statistics of natural images. *Advances in Neural Information Processing Systems*, 12, 855-861.

Appendix

R code for Plotting Probability distributions

```r
#====================================#
# Plots for Distributions
#====================================#

library(fitPlotR)

#====================================#
# Normal Distribution
#====================================#

# PDF of Normal distribution
pdf_norm <- function(x, μ, σ){
  dnorm(x, mean = μ, sd = σ)
}

param_values_norm <- list(
  list(μ = 0.5, σ = 0.5),
  list(μ = 0.5, σ = 1.0),
  list(μ = 0.5, σ = 1.5),
  list(μ = 0.5, σ = 2.0),
  list(μ = 0.5, σ = 2.5),
  list(μ = 0.5, σ = 3.0)
)
plot_pdf(
  pdf = pdf_norm,
  param_list = param_values_norm,
  xlim = c(-5, 5),
  ylim = c(0, 1),
  grid = FALSE
)

# CDF of Normal distribution
cdf_norm <- function(x, μ, σ){
  pnorm(x, mean = μ, sd = σ)
}

plot_cdf(
  cdf = cdf_norm,
  param_list = param_values_norm,
  xlim = c(-5, 5),
  grid = FALSE, main = "CDF Plot"
)

#====================================#
# Gamma Distribution
#====================================#
```

```r
# PDF of Gamma distribution
pdf_gamma <- function(x, α, β){
  dgamma(x, shape = α, rate = β)
}

param_values_gamma <- list(
  list(α = 1, β = 1),
  list(α = 2, β = 1),
  list(α = 3, β = 1),
  list(α = 4, β = 1),
  list(α = 5, β = 1),
  list(α = 6, β = 1)
)

plot_pdf(
  pdf = pdf_gamma,
  param_list = param_values_gamma,
  xlim = c(0, 3),
  ylim = c(0, 1),
  grid = FALSE
)

# CDF of Gamma distribution
cdf_gamma <- function(x, α, β){
  pgamma(x, shape = α, rate = β)

}
plot_cdf(
  cdf = cdf_gamma,
  param_list = param_values_gamma,
  xlim = c(0, 10),
  grid = FALSE, main = "CDF Plot"
)

#====================================#
# Weibull Distribution
#====================================#

# PDF of Weibull distribution
pdf_weibull <- function(x, k, λ){
  dweibull(x, shape = k, scale = λ)
}

param_values_weibull <- list(
  list(k = 0.5, λ = 1),
  list(k = 1.0, λ = 1),
  list(k = 1.5, λ = 1),
  list(k = 2.0, λ = 1),
  list(k = 2.5, λ = 1),
  list(k = 3.0, λ = 1)
```

```
)

plot_pdf(
  pdf = pdf_weibull,

  param_list = param_values_weibull,
  xlim = c(0, 5),
  ylim = c(0, 1.5),
  grid = FALSE
)

# CDF of Weibull distribution
cdf_weibull <- function(x, k, λ){
  pweibull(x, shape = k, scale = λ)
}

plot_cdf(
  cdf = cdf_weibull,
  param_list = param_values_weibull,
  xlim = c(0, 5),
  grid = FALSE, main = "CDF Plot"
)

#====================================#
# Beta Distribution
#====================================#

# PDF of Beta distribution
pdf_beta <- function(x, α, β){
  dbeta(x, shape1 = α, shape2 = β)
}

param_values_beta <- list(
  list(α = 0.5, β = 0.5),
  list(α = 1.0, β = 1.0),
  list(α = 2.0, β = 2.0),
  list(α = 3.0, β = 3.0),
  list(α = 5.0, β = 2.0),
  list(α = 2.0, β = 5.0)
)

plot_pdf(
  pdf = pdf_beta,
  param_list = param_values_beta,
  xlim = c(0, 1),
  grid = FALSE
)

# CDF of Beta distribution
cdf_beta <- function(x, α, β){
  pbeta(x, shape1 = α, shape2 = β)
```

```r
}

plot_cdf(
  cdf = cdf_beta,
  param_list = param_values_beta,
  xlim = c(0, 1),
  grid = FALSE, main = "CDF plot"
)

#====================================#
# Exponential Distribution
#====================================#

# PDF of Exponential distribution
pdf_exp <- function(x, λ){
  dexp(x, rate = λ)
}

# Different values of rate parameter λ
param_values_exp <- list(
  list(λ = 0.5),
  list(λ = 1.0),
  list(λ = 1.5),
  list(λ = 2.0),
  list(λ = 2.5),
  list(λ = 3.0)
)

# Plot PDF
plot_pdf(
  pdf = pdf_exp,
  param_list = param_values_exp,
  xlim = c(0, 5),
  ylim = c(0, 2),
  grid = FALSE
)

# CDF of exponential distribution
cdf_exp <- function(x, λ){
  pexp(x, rate = λ)
}

# Plot CDF
plot_cdf(
  cdf = cdf_exp,
  param_list = param_values_exp,
  xlim = c(0, 5),
  grid = FALSE, main = "CDF plot"
)
```