

## **Building Thinking Classrooms in Mathematics: Learning About Sustained Changes in Teacher Practice through the Interconnected Model of Teacher Professional Growth**

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### **Author's Note**

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### **Abstract**

The Building Thinking Classrooms (BTC) Initiative, a sub-group of a larger Critical Friends Model of teacher professional development (PD) in an urban Manitoba school division, was designed to improve teachers' sense of self-efficacy in mathematics instruction and to promote the adoption of optimal, research-based pedagogical practices. Cohorts of 29-30 primarily math teachers/coaches worked with Dr. Peter Liljedahl (as a critical friend) over a series of six PD sessions each year (one in-person and five virtual) to engage with his research-based classroom practices for enhancing student learning and enact them in their own classrooms. The case-study research described in this paper examined data over two years (2022–2023 and 2023–2024) to look at the impact of the BTC initiative on teaching practice(s), effective elements of the PD model in supporting and sustaining changes in practice, and perceptions of impacts on student engagement, achievement, and self-efficacy. Through the use of Clarke and Hollingsworth's (2002) Interconnected Model of Teacher Professional Growth (IMTPG), change processes were modelled, emphasizing four additional influences on the domains of the teacher's world in the IMTPG, suggesting that contextual and personal influences are critical considerations in the planning of effective PD.

*Keywords:* Building Thinking Classrooms, Interconnected Model of Teacher Professional Growth, mathematics, professional development



## **Building Thinking Classrooms in Mathematics: Learning About Sustained Changes in Teacher Practice through the Interconnected Model of Teacher Professional Growth**

In the 2022–2023 school year, an urban school division in Manitoba, Canada, embarked upon a pilot project to improve teachers' sense of self-efficacy in mathematics instruction, and to promote the adoption of optimal, research-based pedagogical practices in Grades 5–12 mathematics. This project, which was known as the Building Thinking Classrooms (BTC) Initiative, was part of a larger Critical Friends Model of teacher professional development (PD) that brought cohorts of teachers together to work directly with pedagogical experts to learn about classroom strategies for improving student learning. In the case of the BTC Initiative, a cohort of 30 teachers worked with Dr. Peter Liljedahl, a professor from British Columbia, Canada, who wrote the 2021 book titled *Building Thinking Classrooms in Mathematics: 14 Teaching Practices for Enhancing Learning* (Liljedahl, 2021). Over a series of six sessions (dispersed throughout the 2022–2023 school year), teachers engaged with Liljedahl's pedagogical strategies for enhancing student learning in mathematics, trying them out in their own classrooms and debriefing together at their sessions.

Two divisional employees (a Numeracy Specialist and a Continuous Improvement Coordinator) in charge of the BTC Initiative reached out to me in my role as a middle and senior years (Grades 5–8 and 9–12) mathematics methods instructor at a local university to see if I would be interested in partnering with the division to do some research on the project. Because the goal of the BTC initiative was to improve teachers' sense of self-efficacy in mathematics instruction and to promote the adoption of research-based, optimal practices, it was important to assess the impact of the initiative on students and teachers in the division. As a result, the following research questions were collaboratively framed to guide the study:

1. How has the pilot project impacted teachers' classroom practice(s)?
2. To what extent (if at all) has the project improved teachers' sense of efficacy in meeting mathematics outcomes?
3. What elements of implementation of the pilot initiative were effective in changing practice? What elements of the pilot initiative fostered continuation of these changes in practice over time?
4. To what extent (if at all) has the project improved student engagement, achievement, and sense of efficacy in mathematics?

For the purposes of this paper, emphasis will be placed on changes in teacher practice; elements of the initiative that were effective in changing (and sustaining changes in) classroom practice; and perceived impacts on student engagement, achievement, and sense of efficacy.

In addition to the research questions mentioned above, I was curious as a researcher about how this model of PD fits within broader understandings about effective teacher professional development (PD), how it promoted changes in teacher practice(s), and the extent to which such changes in practice were sustained over time. As a result, I utilized both literature from the field on effective teacher PD and Clarke and Hollingsworth's (2002) Interconnected Model of Teacher Professional Growth (IMTPG), a model that illustrates teacher change, to conduct a case study of the BTC Initiative in the hopes of answering the research questions and learning more about effective teacher PD and teacher change.

## Literature on Effective Teacher Professional Development in Mathematics

Many school divisions/districts across Canada (and elsewhere) incorporate professional development initiatives as an approach to improving mathematics teaching and student numeracy skills. Such initiatives frequently involve the introduction of pedagogical (mathematics) strategies and approaches by experienced speakers, researchers, coaches, and the like via presentation or through experiential means in a workshop-type format. The idea is that teachers then incorporate these “new” strategies and approaches in their own classrooms. Literature on effective teacher PD, however, generally does not support initiatives that are “stand-alone” or “one-off” in nature (Campbell et al., 2017; Darling-Hammond et al., 2010; Hardy, 2009). In fact, the elements of effective teacher PD are both well-researched and numerous, as is indicated in Table 1.

**Table 1**

*Characteristics of Effective PD Evident in Literature.*

<b>Characteristics of Effective PD</b>	<b>Supported in Literature</b>
Focused on student learning	Campbell et al. (2017); Guskey (2003); Harwell (2003); Higgins & Parsons (2009); Hunzicker (2010a); Learning Forward (2011); Mundry (2005); Murray (2014); Reeves (2010); Skyhar (2018, 2020); Timperley (2008); Whitcomb et al. (2009)
Includes subject-specific content and pedagogical content knowledge	Bredeson (2002); Campbell et al. (2017); Harwell (2003); Higgins & Parsons (2009); Hunzicker (2010a, 2010b); Mundry (2005); Murray (2014); Porter et al. (2003); Quick et al. (2009); Skyhar (2018, 2020); VanDriel & Berry (2012)
Aligned with school, district, curricular, and individual teacher goals	Bredeson (2002); Campbell et al. (2017); Hunzicker (2010a, 2010b); Murray (2014); Porter et al. (2003); Quick et al. (2009); Skyhar (2018, 2020)
Opportunities for active learning	Campbell et al. (2017); Darling-Hammond & McLaughlin (2011); Hunzicker (2010b); Porter et al. (2003); Quick et al. (2009); Skyhar (2018, 2020); Timperley (2008); Villegas-Reimers (2003)
Collegial and collaborative learning environment characterized by respect, trust, safety, and accountability	Bredeson (2002); Bruce et al. (2010); Campbell et al. (2017); Darling-Hammond & McLaughlin (2011); Goos et al. (2011); Hargreaves & O’Connor (2018); Harwell (2003); Hunzicker (2010a, 2010b); Learning Forward (2011); Murray (2014); Nelson et al. (2010); Porter et al. (2003); Quick et al. (2009); Skyhar (2018, 2020); Timperley (2008); VanDriel & Berry (2012); Whitcomb et al. (2009)
Embedded in the daily life of schools	Bredeson (2002); Bruce et al. (2010); Campbell et al. (2017); Darling-Hammond & McLaughlin (2011); Goos et al. (2011); Hunzicker (2010a, 2010b); Mundry (2005); Murray (2014); Quick et al. (2009); Skyhar (2018, 2020)

Ongoing in duration	Campbell et al. (2017); Darling-Hammond & McLaughlin (2011); Harwell (2003); Hunzicker (2010a, 2010b); Murray (2014); Porter et al. (2003); Quick et al. (2009); Skyhar (2018, 2020)
Scalable and sustainable	Loucks-Horseley et al. (2010); Skyhar (2018, 2020); Timperley (2008); Whitcomb et al. (2009)
Adequate support in terms of time, resources and leadership	Bredeson (2002); Campbell et al. (2017); Darling-Hammond & McLaughlin (2011); Goos et al. (2011); Hunzicker (2010a, 2010b); Learning Forward (2011); Mundry (2005); Quick et al. (2009); Skyhar (2018, 2020); Timperley (2008); Villegas-Reimers (2003)

Given the characteristics of effective teacher PD listed in Table 1, it follows that effective teacher PD in mathematics should be not only aligned with divisional goals of improving student numeracy skills but also focused specifically on mathematics content and pedagogy as well as student learning. In addition, effective teacher PD in mathematics should include ongoing, job-embedded, active PD opportunities in which teachers are able to come together in a collegial and collaborative environment to learn and dialogue about mathematics, mathematics teaching, and student learning. Such experiences should occur throughout the year and include adequate time and resources for teachers to embed their learning within their own classroom contexts.

### The BTC Initiative

The BTC Initiative embodied most (if not all) of the characteristics of effective teacher PD identified in Table 1. The initiative brought together 30 volunteer teachers/coaches, comprised of 4 Senior Years (Grades 9–12) teachers, 18.5 Middle Years (Grades 5–8) teachers, 6.5 Academic/Numeracy Support Teachers, and 1 SY Continuous Improvement Coach (note that some teachers had dual roles and were counted as 0.5 according to their dual roles). The cohort was co- led by two divisional employees, a Numeracy Specialist and an Administrator of Continuous Improvement, and met six times over the course of the 2022–2023 school year for full-day (9 am – 3 pm) PD sessions with Peter Liljedahl (one in-person in the spring and 5 virtual). At these sessions, Liljedahl, as a critical friend, shared pedagogical practices (actively and experientially) with cohort teachers/coaches from his research and the book published about it. Teachers/Coaches were provided with his book, vertical non-permanent learning surfaces (VNPSs) in the form of laminated white sheets (Wipebooks), and dry-erase markers and encouraged to try out his pedagogical strategies in their own classrooms between sessions. They were then allowed to reflect on their experiences when they met, before learning about more pedagogical practices for implementation. As a result, the BTC Initiative was ongoing in duration; focused on mathematics, mathematics pedagogy and student learning; aligned with divisional numeracy goals; and provided time, resources, and leadership. Teachers worked within a collaborative cohort, engaged in active online and in-person experiences with Liljedahl, and worked to implement strategies within the contexts of their own classrooms (job-embedded). Finally, the cohort model allowed for the initiative to be scaled up and sustained over time, as more teachers joined cohorts, creating a critical mass of teachers in the division that had experienced Liljedahl’s practices for enhancing student learning.

## Liljedahl's Pedagogical Practices for Enhancing Learning

In his 2021 book, Liljedahl describes how he noticed in his observations of teaching and learning that students weren't thinking, and teachers were planning their teaching based "on the assumption that students either couldn't or wouldn't think" (p. 6). This is what spurred him to spend the next 15 years working with "over 400 K–12 teachers to try to break through the non-thinking behaviours and get students to think" (p. 12). What ensued was a series of classroom experiments with teachers, searching for "local optimal practices" (p. 15) that could be scaled up to work for any teacher. Weeks were spent fine-tuning each "optimal practice for thinking," and the result was a book with a chapter on each of fourteen strategies that impact thinking in a classroom.

Of the optimal practices described in the book, one of the most significant changes to traditional mathematics teaching is the use of VNPSs. Liljedahl proposes in the book that teachers decenter the classroom and have students work standing at VNPSs in groups of three. This, according to Liljedahl, promotes thinking and engagement and decreases reliance on the teacher. Tasks are given to students shortly after entering the room, and rather than modelling through showing students how to solve problems (e.g. at the front of a whiteboard or via PowerPoint) and expecting them to mimic procedures (e.g. on practice questions), teachers require students to approach novel problems at VNPSs that are open and easily extended. Once students are working, teachers only provide hints and extensions needed to help students think, as opposed to answering questions about how to solve the problem or verifying that students have the correct answer. Learning is consolidated in such an approach by looking at the work of various groups, noting commonalities and unique approaches, and having students explain their thinking and the thinking of others. Finally, notes and homework, frequently used in traditional classrooms, are optimized in Liljedahl's 'thinking classrooms' as well, as they are personalized and suggested (not required) for students, giving them autonomy and choice in how they document and solidify their learning.

### The Six PD Sessions

The six sessions that were led by Liljedahl broke his fourteen practices down into smaller chunks known as "toolkits" (see Table 2 below).

**Table 2**

*BTC Toolkits.*

<b>Toolkit 1</b>	Thinking tasks, random groupings, vertical non-permanent surfaces (VNPSs)
<b>Toolkit 2</b>	Defronting the classroom, answering only keep thinking questions, give thinking tasks (early, standing, verbally), CYU questions, mobilize student knowledge
<b>Toolkit 3</b>	Use hits and extensions to maintain flow, consolidate from the bottom up, have students write meaningful notes
<b>Toolkit 4</b>	Evaluate what I value, help students see where they are and where they are going, grade based on data not points

Sessions tended to focus on one toolkit (and section of chapters in the book) at a time, allowing Cohort teachers to be introduced to them, work with Liljedahl, and then put them into practice in their classrooms in between sessions. In this way, the practices were gradually introduced over the course of the school year with a focus on the implementation of new learning

in a practical setting. By the end of the first year of the BTC Initiative, all 30 teachers had worked through all four of the toolkits with Liljedahl and been encouraged to try the practices in their own contexts.

## Year 2 of the BTC Initiative

In 2023-2024, two groups essentially emerged in relation to the BTC initiative: the Original Cohort, which consisted of 8 MY teachers/coaches of the original 30 who continued to meet with the Numeracy Specialist to work on implementing Lildjedahl's pedagogical practices; and the Secondary Cohort, a new group of 29 teachers who started a new series of 6 sessions (1 in-person in the fall and 5 virtual) with Liljedahl. While the content of the Secondary Cohort sessions with Liljedahl was similar, two important changes in structure were made in response to feedback provided by the 2022-2023 Cohort. The first was that the spring in-person session with Liljedahl was moved to the fall in order to 'see' Liljedahl's strategies enacted (by him, in-person) sooner in the year. The second important change in structure for the Secondary Cohort was that time was built into the afternoons of two of the sessions to debrief and unpack what had been said, and to plan for classroom implementation. This was also done in response to teacher feedback.

In terms of the meetings held for the Original Cohort in the second year, a total of three full-day sessions were held in 2023–2024, led by the Numeracy Specialist. During these sessions, participants reviewed the fourteen BTC practices (including micro moves), reviewed new BTC research, shared task resources, and created tasks, navigation instruments, and check-your-understanding (CYU) questions. Participants had the opportunity to share how things were going in their own classrooms, ask questions of their colleagues and the Numeracy Specialist, and hear about new things shared by Liljedahl at the Secondary Cohort meetings. They were also afforded much-needed time to work collaboratively on resource development.

## Research Methods

A single case study design, an appropriate methodological choice for an in-depth study of a single unit or bounded system (Creswell, 2007; Flyvbjerg, 2011; Merriam, 1998; Stake, 1995), was utilized for this study. The BTC project provided a bounded case that was both unique and revelatory (Yin, 2009), allowing for insight to be gained into the approach to professional development taken, its impact on the instructional practices of divisional teachers, and the resulting influence of the project on student engagement, achievement and sense of self-efficacy in mathematics (note that one teacher also used the strategies in a Physics context). In order to gain such insight, both primary and secondary data were collected over the first two years of the initiative (see Table 3 below).

**Table 3**

*Data Sources.*

Primary Data (Year 1)	Primary Data (Year 2)	Secondary Data Provided by Division
6 teacher/coach interviews: 1 high school teacher, 4 middle years teachers and 1 middle years teacher/coach	4 teacher interviews: 1 high school teacher and 3 middle years teachers (3 were interviewed in previous year, 1 was not)	Financial information

3 student focus group conversations with students of teachers/coaches interviewed: 1 high school and 2 middle years focus groups (4-5 students each)		Notes and summaries of activities from Numeracy Specialist & Continuous Improvement Administrator
2 interview(s) with pilot project co-leads - Numeracy Specialist and Continuous Improvement Administrator	2 interview(s) with pilot project co-leads - Numeracy Specialist and Continuous Improvement Administrator	Anonymous survey data from teachers participating in the pilot (after Year 2)

Teachers/Coaches and co-leads were recruited to participate in semi-structured interviews for the study in both the first and second year via email (with attached letter of invitation). Teacher/Coach semi-structured interviews focused on questions about what their practice looked like prior to the BTC Initiative, what it looked like after, effective elements of the PD initiative, impacts on their sense of efficacy as teachers, and perceived impacts on student engagement, achievement, and self-efficacy. Co-lead semi-structured interview questions focused on elements of the initiative and perceived impacts on teachers' practice(s) and student engagement, achievement, and self-efficacy in mathematics.

Students of the teacher interviewees in the first year were recruited through the use of a physical letter of invitation to parents and students. All participants (and their guardians if under 18 years of age) signed a consent form prior to participation. Focus group questions with students focused on perceived changes in their teachers' practices and their own perceptions of their self-efficacy as math students, their achievement in the class, and their engagement with the new pedagogical and learning strategies experienced.

Interviews and focus group discussions were audio-recorded and transcribed for analysis, and all interview participants were provided with copies of their transcript(s) for editing/adjusting and verification prior to its/their use as data for the study (member-checking).

Secondary data in the form of reports, notes, files, summaries and survey data were also provided annually by the co-leads of the initiative following their individual interviews. The school division approved both the study and the forms of data collected prior to the commencement of the research, as well as all changes made to the research methods and protocols in the second year.

The first round of data analysis utilized reflective thematic analysis, a process involving the following six phases: 1) familiarizing yourself with the data set, 2) coding, 3) generating initial themes, 4) developing and reviewing themes, 5) refining, defining, and naming themes, and 6) writing up the findings (Braun & Clarke, 2022, p. 35-36). Following transcription and member-checking of interviews and focus group conversations each year, the researcher familiarized herself with the data and performed initial coding (Saldaña, 2009) using topics related to literature about effective teacher PD and the collaboratively designed research questions. Coding was also extended to include emergent themes from the data before codes and themes were refined into broad categories or themes. A second round of coding and data display also took place, utilizing charts/tables and Clarke and Hollingsworth's (2002) IMTPG model, as changes in practice for

each teacher were organized into categories and an overall trajectory of teacher change was displayed using the model.

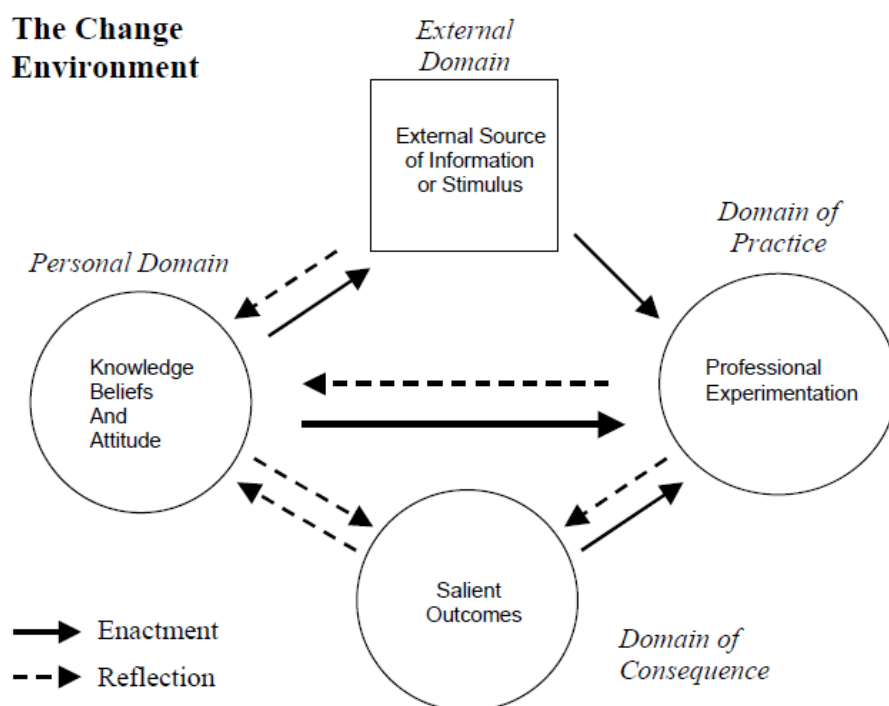
### Theoretical and Conceptual Frameworks

The research conducted draws on social constructivist theory, which acknowledges that individuals (including teachers engaging in PD) construct understandings of new phenomena as they engage in actions (experiences, activities, dialogue and reflection) that allow new ideas to rub up against existing understandings, beliefs and attitudes (Richardson, 1997, 1999). All of this happens within a social context that cannot be separated from the individual learning that occurs (McCullagh, 2012; Pitsoe & Mailia, 2012; Richardson, 1997, 1999). Within the context of mathematics teachers engaging in PD as cohorts, this means that the actions they engage in together, including the new information they are exposed to, the conversations they have together, and the experiences have in their own classrooms and then unpack together in sessions, foster the individual construction of new understandings, beliefs, and attitudes that are influenced in complex ways by the social context in which the learning takes place.

As a conceptual model, Clarke and Hollingsworth's (2002) IMTPG model illustrates the complex process of social construction of new understandings, beliefs, and attitudes through the process of 'enactment' and 'reflection' across the four domains of the teacher's world: the external domain, the personal domain, the domain of practice, and the domain of consequence (see Figure 1). As a result, it is a useful tool for both thinking about teacher change and for looking at data for evidence of this change.

**Figure 1**

*The IMTPG (Clarke & Hollingsworth, 2002, p. 951)*



## Findings

Findings from the study are broken down into five sections: changes in the classroom practices of teachers; elements of the initiative effective in changing and sustaining changes in practice; barriers to affecting changes in practice and suggestions from teachers; illustrating teacher change with the IMTPG, and impacts on student engagement, achievement and efficacy. These sections align with the collaboratively designed research questions that guided the study. The findings from the study are also followed by a discussion of their importance in terms of what is known about effective teacher PD and teacher change.

### Changes in the Classroom Practices of Teachers

After the second year of the BTC initiative, a survey was sent out to all participants in the BTC initiative by the Numeracy Specialist. Of the 59 participants in the initiative, 41 responded to the survey. The survey focused on gathering information about the degree to which participants planned to implement the 14 BTC practices outlined by Peter Liljedahl in the future. Of the respondents, 35 planned on implementing the fourteen practices in the 2024-2025 school year, although to varying degrees (19 indicated they would implement *some* of the practices, 10 indicated they would implement *most* of the practices, and six indicated they would implement *all* of the practices). Six of the respondents indicated that they would not be implementing the practices in the following year (five of these were no longer teaching math/their teaching assignment had changed, and one felt they did not have enough time to adapt their lessons to the BTC style, given their other commitments).

In addition to the survey data, interviews conducted with participants in both years of the study further explained what classroom changes had taken place for the Original Cohort (within broad thematic categories identified through the coding process). In particular, the trajectory of four of the participants highlighted the varying levels of changes that took place over 2022-2023 and 2023-2024. Participant 1, for example, implemented many of the 14 practices consistently, beginning prior to the BTC Initiative and extending through both years. In addition, they utilized their own assessment methods, opting to use an ‘ungrading’ strategy in their high school physics courses.

**Table 4**

*Participant 1 (High School Physics/Math) – Interviewed in both years.*

	Prior to BTC	Year 1	Year 2
<b>Classroom</b>	Physics lab with immovable tables. Smartboard at front.	Room remained the same with the addition of VNPSs and “scientist” labels for random groupings.	
<b>Instruction</b>	Fairly traditional. Examples on Smartboard at front: “I do, We do, You do”	Started before BTC initiative. Utilized visibly random groups, moved between Smartboard and VNPS.	Continued to use VNPSs and visibly random groups daily.

<b>Student work</b>	Individual whiteboards used during lesson. Independent work after board work. Cumulative exercise packages created by teacher (20 problems) – Do every odd one, even if nec.	Math - Used problem sets to create VNPS tasks. Students did them collaboratively at VNPSs.  Physics – a couple of scaffolded problems (would previously have done as examples on board) done at VNPSs.	Not teaching math.  Physics – a couple of scaffolded problems (would previously have done as examples on board) done at VNPSs.
<b>Clarifying/ Consolidating</b>	Go over questions the next day related to the problems.	Worked on consolidation at VNPSs after seeing Liljedahl in person.	Improving consolidation skills at VNPSs.
<b>Notes</b>	Notes packages.	Tried out Liljedahl’s methods for note-taking.	Liljedahl’s four corners note-taking method.
<b>Homework</b>	Not assigned. Up to student to do as many questions/problems as needed.	Math – doing problems at the VNPS and more specialized problems as independent work.  Physics – 3-4 specialized problems (attempt from multiple perspectives).	No longer teaching math.  Physics – 3-4 specialized problems (attempt from multiple perspectives).
<b>Assessment</b>	Moved from traditional tests (written, outcomes-based) to ‘ungrading’ (portfolios, interviews, skills-based, students self-assess and negotiate grade with teacher). Mostly in physics. Math remained more traditional.	Continued with ungrading in physics and traditional tests in math.	No longer teaching math.  Continued with ungrading in physics.
<b>Planning</b>		Math – Used the cumulative exercises as a starting point for thin slicing tasks.  Physics – Used previous examples and materials to create tasks.  Also consulted a lot of websites and online resources.	

Participant 5, on the other hand, had limited implementation of BTC strategies in their Grade 7 classroom. While they were able to dabble with the strategies in the first year and work collaboratively with a Coach at the beginning of Year 2 of the initiative (about six weeks), competing PD initiatives (ELA) and changes in personnel with whom they were collaborating made it difficult to fully implement the BTC practices. Participant 5 continued to use the VNPSs in their classroom; however, primarily for review purposes.

**Table 5**

*Participant 5 (Grade 7) – Interviewed in both years.*

	Prior to BTC	Year 1	Year 2
<b>Classroom</b>	Desks facing front or table groups (with defined front).	Defronted – no table groups facing the front (facing all directions).	
<b>Instruction</b>	First 20 minutes – mental math game. Next 20 minutes – Teacher introduces new concepts.	Tried the VNPSs approximately 10 times over the course of the year. This involved a 5-minute introduction followed by 30-40 min. of VNPS work before coming back together to consolidate.	Began the year partnering with a numeracy specialist for 6 weeks. Began with non-curricular tasks and extended into curricular tasks. Tried a thin slicing lesson in October.  Continued use of VNPSs for Review.
<b>Student work</b>	20 minutes – students work independently on practice.		Utilized Mild/Medium/Spicy with VNPS work in reviews.
<b>Clarifying/ Consolidating</b>			
<b>Notes</b>			
<b>Homework</b>	These were not discussed in the interviews.		
<b>Assessment</b>			
<b>Planning</b>			When no time to plan, used the banner (at VNPSs) and reviewed concepts.

Participants 6 and 10 became strong collaborative partners in Year 2. While Participant 10 had their position change partway through Year 1 of the initiative, they returned to a Grade 5/6

classroom in Year 2 and partnered with Participant 6 to fully implement all Liljedahl's toolboxes. Participant 6 had the experience of moving from Grade 5 to Grade 6 over the two years (8 out of 24 students made the move with them). Participants 6 and 10 met weekly on Tuesdays after school to plan together and spent a lot of their own time on evenings and weekends creating materials for use in their classrooms (which they shared both with each other and to a lesser extent at the Year 2 sessions that met 3 times in 2023-2024). Both teachers began their second year fully implementing Liljedahl's strategies, having tried several of them out in their first year in the BTC project.

**Table 6**

*Participant 10 (Grade 5 /6) – Only interviewed in Year 2.*

	Prior to BTC	Year 1	Year 2
<b>Classroom</b>			Room defronted. Desks in groups facing different directions. Whiteboards around classroom.
<b>Instruction</b>	Instruction at the front (PowerPoint).  Used manipulatives.  Individual whiteboards.	Tried lessons with VNPSs a handful of times.  Was pulled out of classroom halfway through the year.	Full implementation of Liljedahl strategies from first day of the year. Almost every day (tasks, thin slicing). Worked a lot on routines and expectations (also employed video for this). 10 days of non-curricular tasks and then into place value. Used VNPSs in other subjects as well.
<b>Student work</b>	Individual whiteboards.  Paper-pencil tasks.  Used manipulatives.	.	Daily at the boards (used cards with answers on the back) and then levelled choices (CYU) in their notebooks (mild, medium, spicy). Focus on justifying thinking. Finds the notebook provides lots of information about student understanding.
<b>Clarifying/ Consolidating</b>			Consolidated at VNPSs and then moved to CYU questions.
<b>Notes</b>			Implemented notes to your future forgetful self for each topic in the unit. Started out

		fairly guided in the beginning, becoming more independent.
<b>Homework</b>		
<b>Assessment</b>	Written assessments.	Used the CYU in notebooks in addition to 1–2-page written assessments. Students allowed a notes page/cheat sheet for written assessments.  A lot of work (self/group) assessing norms and expectations at VNPSs and of work.
<b>Planning</b>		Worked with Participant 6. They met once a week (Tuesdays) throughout the year to plan together plus spent several hours on their own creating tasks and materials.  Participated in Year 2 group of 7-8 participants (3 meetings over the year).

**Table 7**

*Participant 6 (Grade 5 Year 1, Grade 6 Year 2) – Interviewed in both years.*

	<b>Prior to BTC</b>	<b>Year 1</b>	<b>Year 2</b>
	<b>Grade 5/6</b>	<b>Grade 5</b>	<b>Grade 6</b>
<b>Classroom</b>		Groups/Centers/Clusters of desks defronted.	
<b>Instruction</b>	Centers used – each group has a math menu with 2-ish must do activities for a 30 min. class.  Lessons taught through center groups or full class.	Continued with centers (but not math menus).  Began to implement lessons with VNPSs and CYU questions (mild, medium and spicy). (2X per week).	8/24 students experienced VNPSs the previous year with the teacher.  Full implementation of Liljedahl strategies from first day of the year. Almost every day. A lot of

Tried some thin slicing and banners. thin slicing and larger curricular tasks at VNPSs.

<b>Student work</b>	Math Menus – ‘Must do’ activities (lesson with the teacher, find the error, etc.) and ‘may do’ activities once finished (e.g. sudoku, Esti-mysteries, which one doesn’t belong, etc.). Collaborative or independent.	Began to implement Check Your Understanding (CYU) questions with levels – mild, medium and spicy.	Daily at the VNPSs and then levelled choices (CYU) in their notebooks (mild, medium, spicy), noted this helped identify struggling students and misconceptions.  Also used Fullerton’s Big 4 this year – 4 big questions on the topic (not leveled).
<b>Clarifying/ Consolidating</b>		Started some consolidation when trying VNPS tasks.	Consolidated at VNPSs and then moved to CYU questions.
<b>Notes</b>	Teacher directed.	Used scaffolded notes where notes were given and students found errors in problems. Also had students find their own examples.	Implemented notes to your future forgetful self for each topic in the unit. Started out fairly guided in the beginning, becoming more independent. Employed Liljedahl’s four corners/quadrant note-taking method.
<b>Homework</b>	Did not assign homework		
<b>Assessment</b>	Written assessments for each grade (5 and 6).	Tried outcomes-based learning maps/checklists suggested by Liljedahl.  Started to include leveled questions (mild, medium, spicy) on assessments.	Continued with leveled questions on written assessments. Students allowed a notes page they made on blank template.  Students self-assess their group work skills.

## Planning

Worked with Participant 10. They met once a week (Tuesdays) throughout the year to plan together plus spent several hours on their own creating tasks and materials.

Participated in Year 2 group of 8-9 participants (3 meetings).

Overall, these tables illustrate the varying degrees of implementation and change among teachers in the BTC initiative. Some teachers dabbled in using the strategies, while others engaged in sustained, ongoing growth and change over the 2022–2023 and 2023–2024 school years.

### **Elements of the Initiative Effective in Changing and Sustaining Changes in Practice**

Participants in the study identified many elements of the initiative that were effective in changing practice and supporting student learning, and that contributed to the continuation of changes in practice over time. Included in the effective elements were several factors related to support in the working environment, such as the support provided by peers (other Cohort teachers/coaches), the Numeracy Specialist, Numeracy Coaches in the division, administrators, and Liljedahl, himself (particularly when in-person). Additionally, participants found working in an environment with like-minded colleagues (colleagues in the same school or who were also part of the Cohort) especially beneficial (at sessions and in the Year 2 follow-up), although they noted that having a personal commitment to the process, including a willingness to spend hours of personal time to create tasks, plan and collaborate, was also important. Overall, teachers liked the accessibility of the online sessions, despite a few technology issues, and noted that the ongoing nature of the PD promoted change as is evident in the following statement: “I did like the idea of it being like a yearlong, kind of ongoing, ongoing process, as opposed to like, doing it one day, and then kind of, you know, forgetting about it, almost” (P5 Interview, Year 2). Finally, one of the participants noted that elements of crossover between other Critical Friends Model PD sessions (specifically one focused on developing an understanding of the ‘numerate learner’) were also helpful.

### **Barriers to Affecting Changes in Practice and Suggestions from Teachers**

In addition to identifying effective elements of the model, teachers/coaches also identified several challenges/barriers to affecting and sustaining changes in practice. For example, Participant 1 noted that high school teachers were not included in the three Year 2 follow-up sessions offered to middle years teachers in the Original Cohort (minimizing their ability to continue with the support of colleagues). In addition, while participants understood the financial constraints in the division, they noted that it was much more beneficial to work with Liljedahl in person than virtually, due to technology glitches and the isolating nature of working online. Several participants, including the Numeracy Specialist and Continuous Improvement Coordinator, also acknowledged varying levels of buy-in from teachers, attributing this to a variety of factors such as complex classroom/working environments (e.g. challenging student behaviors, background gaps due to COVID, heavy

workloads, lack of substitute teachers to attend PD sessions, competing PD initiatives and service work, lack of time to collaborate with others, lack of access to colleagues in the same building to plan with, and lack of BTC-related resources/tasks), and individual teacher factors (e.g. lack of personal time to plan, and level of personal commitment, including a tendency to revert to traditional methods).

In analyzing the changes in practice evident in the interview data and the effective elements and barriers described by teachers alongside the literature on effective teacher PD, it is evident that the BTC Initiative (overall) aligned very well with many characteristics of effective PD. In particular, its alignment with the division's numeracy goals and other initiatives in the division (teachers spoke about crossover) was effective for teachers, as were both the collaboration with peers (or like-minded colleagues) and the ongoing nature of the PD (six sessions over the course of the year). Even though working with Liljedahl online posed some technical difficulties, the many sessions over the course of the year, supported by the leadership of the Numeracy Specialist and Continuous Improvement Coordinator, were described as effective by teachers consistently. As discussed in the next section, teachers also found the strong focus on student engagement and learning alongside the specific pedagogical strategies (actively) presented particularly effective, aligning with what is known about effective PD as well; and although teachers didn't explicitly speak about the scalability of the initiative, some were able to sustain significant changes in practice over time. Interviews with the Numeracy Specialist and Continuous Improvement Coordinator also described the division's plan to scale up the initiative at length, indicating that there were strong elements of scalability and sustainability built into the model.

Despite very strong alignment with what is known about effective teacher PD, there was still variance in terms of both the depth of changes in practice seen and the degree to which changes were sustained over time. These differences largely reflected individual circumstances (personal and/or in terms of classroom/school/divisional context). Table 8 outlines four primary influences on teachers' ability to make changes in practice and/or to sustain changes in practice over time (along with descriptions of how they were evident in the study data).

**Table 8**

*Influences on Teacher Change.*

<b>Influence</b>	<b>Description</b>
<b>Personal Time</b>	The two teachers (Participants 6 and 10) who engaged in robust growth and implementation of the Liljedahl strategies in year 2 of the study spent an incredible amount of their own personal time to keep the cycles of growth going. Teachers with small children at home or without the personal time to devote to this growth may not have been able to continue as effectively over time.
<b>Personal Commitment</b>	Some teachers in the project did not have a high level of commitment to the strategies they were exposed to. The reasons for this were varied (e.g. skepticism about the research, concern about student behaviour, being 'voluntold' to participate in the initiative rather than volunteering themselves). Those teachers who didn't have a high level of commitment were not able to continue with the strategies effectively over time.

**Classroom Environment**

Changes in teaching assignments altered teachers' ability to engage in ongoing, sustained change, as did the students and student behaviours. Teachers made decisions about how much to implement the practices based on perceived student needs. For example, one teacher interviewed in the first year (Participant 7) elected to use the strategies only for enrichment. In addition, positive student experiences (e.g. students in FG 2 who were the students of Participant 1) also fostered continuation of teacher changes in practice over time.

**School/ Divisional Environment**

The broader environment in the school/division also impacted what teachers were able to manage in terms of changes in practice. For example, competing initiatives impacted the Domain of Practice for some teachers (e.g. Participant 5) overtaxing their capacity for engaging in PD, for being away from the classroom, and for implementing new classroom strategies. This interrupted networks of growth and/or made ongoing growth difficult. Other divisional initiatives (e.g. PD related to developing a vision of the numerate learner) fed into the growth network as additional external information/stimulus, thereby fostering change/growth.

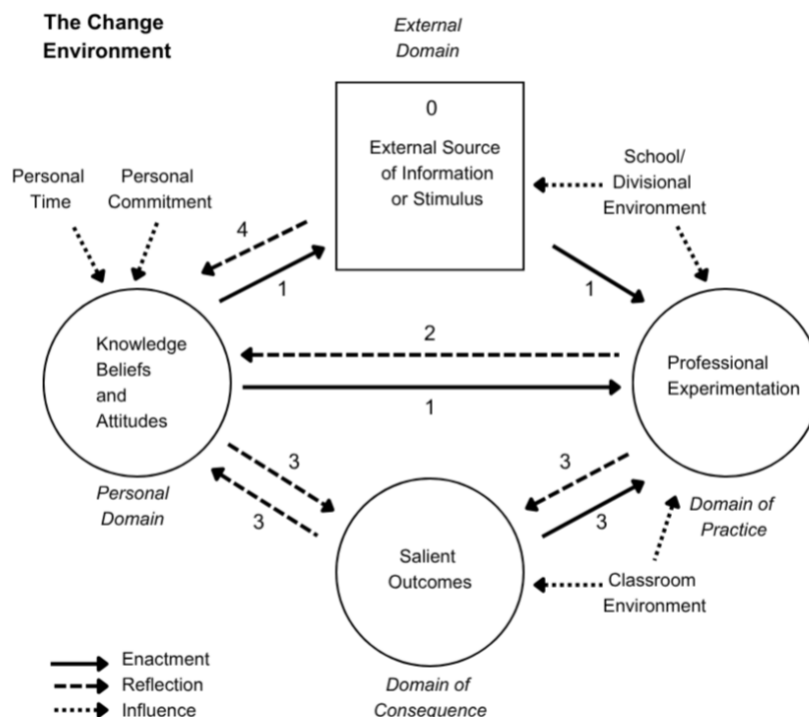
In terms of elements of effective teacher PD, these four influences impacted aspects of several of the elements, including: alignment with teacher goals, teacher accountability, the sustainability of both changes in practice and engagement with the PD/Initiative, and the amount of time that was available for ongoing learning. Such impacts decreased the effectiveness of the PD for individual teachers.

**Illustrating Teacher Change with the IMTPG**

As part of the second round of data analysis, Clarke and Hollingsworth's (2002) IMTPG Model was used to model/illustrate teacher changes in practice as described in the study data. The characteristics of the BTC Model, including the elements of effective PD it incorporated in its design, allowed for significant changes in teacher practice to take place; for improvements in student engagement, achievement, and sense of efficacy to occur; and for growth in teacher knowledge, beliefs and values to transpire. Such a change, along with the influences described in Table 8, could be illustrated using the IMTPG model as follows:

**Figure 2**

*Changes in the BTC Initiative as Illustrated Using the IMTPG Model*



The numbers superimposed on the IMTPG model in Figure 2 illustrate the change process (as evident in the study data) through what Clarke and Hollingsworth (2002) call *change sequences* and *growth networks*. Change sequences, according to Clarke and Hollingsworth (2002), can be described as follows:

A change sequence consists of two or more domains together with the reflective or enactive links connecting these domains, where empirical data support both the occurrence of change in each domain and their causal connection. A change in one domain may not lead to a change in another. Where it does, we employ the term “change sequence.” Such a change may be fleeting, a single instance of experimentation, quickly relinquished. (p. 958)

Participant 5’s experience of trying out Liljedahl’s strategies but relinquishing them except for during unit reviews is an example of a change sequence in which the *Domain of Practice* was temporarily changed, but in which the *Personal Domain* and *Domain of Practice* were not changed long-term. Growth networks, however, are more lasting in nature, according to Clarke and Hollingsworth (2002), and can be described in the following way:

The term “growth” is reserved for more lasting change. This does not preclude a changed practice or belief from being further adapted or refined. Indeed, the adoption of a growth perspective conceives of change as ongoing. Where data have demonstrated the occurrence of a change that is more than momentary, then this more lasting change is taken to signify professional growth. A change sequence associated with such professional growth is termed a “growth network”. (p. 958)

In the case of participants in the BTC Initiative (e.g. Participant 6 and 10), growth networks (through the processes of *enactment* and *reflection*) could be described by the numbers superimposed on Figure 2 as follows:

0. Participant reads Liljedahl’s book and attends initial PD session(s)
1. Participant reflects on own knowledge, beliefs and attitudes about effective mathematics teaching and learning and implements the first toolkit in the classroom (utilizing both own understandings and the information gathered from Liljedahl’s work).
2. Participants reflect on what they experience as they enact the toolkit, informing their knowledge, beliefs and attitudes about effective mathematics teaching and learning.
3. Participant observes/assesses student engagement and understanding of curricular outcomes, which impacts their next steps in practice and their knowledge, beliefs and attitudes about effective mathematics teaching and learning.
4. Participant accesses further information through additional PD sessions, own research, discussions with other teachers, collaboration, etc., and the cycle begins again (with new information/toolkits/knowledge)

It is important to note that the strongest growth networks, like the one illustrated by the number sequence in Figure 2, included accessing external information in a cyclical, ongoing way. It was important in the BTC project to not only have PD sessions over a year, but for teachers to continue learning through their own research, planning, discussion and reflection. The ongoing nature of the BTC initiative, along with its cyclical integration of new toolkits, fostered multiple, interrelated change sequences that created an environment in which growth networks could emerge. For those teachers who were able to continuously engage (e.g. Participant 6 and 10) over the two years (including spending significant amounts of their own time in between sessions), deep changes in practice resulted. For those who didn’t have ongoing support for reflection and enactment in the second year (e.g. Participant 1), changes in practice were sustained but didn’t necessarily grow further. And for those who were not able to continuously engage in cycles of enactment and reflection (e.g. Participant 5), little lasting change was possible. Personal and contextual influences, such as commitment to the initiative, personal time available outside of the sessions to continue the work, and classroom/school/division contexts, played a significant role in the robustness and sustainability of growth networks that supported lasting change.

### **Student Perceptions of Impacts on Engagement, Achievement and Efficacy**

Impacts on student engagement, achievement and sense of efficacy in mathematics/science were evident in both student focus group comments and in the teacher/coach interviews. Table 9 (below) summarizes key themes found in student comments from the focus group discussions.

**Table 9**

*Student Comments Related to Engagement, Achievement and Efficacy.*

	<b>Summary</b>	<b>Sample Student Comments</b>
<b>Engagement</b>	Students found the BTC strategies engaging.	“When she first told us we were going to start working at the whiteboards and stuff, most of the people in our class didn't think it was going to be fun or anything. And then when we started it, it gets kind of fun. Like if she said, “No more whiteboards; We're not

	<p>The BTC strategies fostered connection between students.</p>	<p>doing whiteboards anymore,” I don't think any of us in this class would be happy” (Student 1, FG 1).</p> <p>“Booklets are boring, whiteboards are not boring” (Student 3, FG 1).</p> <p>“Yeah, because like it's generally like associated with a hard course, but [teacher] makes it fun and easy” (Student 1, FG 2).</p> <p>“It's [check your understanding questions] optional, so I'll do it. I don't know. It just makes me want to do it more” (Student 1, FG 2).</p> <p>“It's fun!” (Student 3, FG 3).</p> <p>“I just really like people telling me why they did certain things. You get to learn your board mates’ . . . strengths and weaknesses” (Student 2, FG 3).</p> <p>“Everyone in the class mostly knows each other by this time because there is not one person you haven't . . . sat down beside in class, so it just helps the entire class just like get to know each other” (Student 3, FG 2).</p>
<p><b>Achievement</b></p>	<p>Students’ comments about achievement were inconsistent. Some students (usually with high marks already) felt their marks were the same. One student in Physics noted Physics was their lowest mark. Some students felt they were doing better.</p>	<p>“Well, I always get the same 100%. . . But I study at school and also at home” (Student 5, FG 1).</p> <p>“My physics has, I'd say the same as last semester” (Student 3, FG2).</p> <p>“I have the lowest mark [92%] from all of my classes in physics class” (Student 4, FG 2).</p> <p>“I was mediocre at math, and I feel like I'm kind of better now” (Student 1, FG 1).</p> <p>“I used to be okay at math and now I'm a lot better [up 15-20%]” (Student 3, FG 1).</p> <p>“I feel like I'm doing way better than in the last physics class” (Student 2, FG 2).</p> <p>“[I did] 100% better. Doing that on a piece of paper was just a pain” (Student 4, FG 3).</p>
<p><b>Efficacy</b></p>	<p>Students felt safer to take risks in front of their peers.</p> <p>Students indicated that working with their peers helped</p>	<p>“You can trust the other person to criticize you. You're open to their criticism, which is nice because now you know that, oh, I'm wrong and now I know the right thing” (Student 1, FG 3).</p> <p>“Because then if you make a simple mistake and they realize, then you don't get it wrong because you just made a mistake. Then they can correct you and help you learn it better” (Student 2, FG 3).</p> <p>“More working with friends. If you can try your best, you make mistakes, there's people who help you” (Student 5, FG 1).</p> <p>“We just learn better together” (Student 3, FG 1).</p>

them learn math/science.

Students felt capable of problem solving/engaging in thinking tasks in their classrooms.

“Honestly, we can figure this out together” (Student 2, FG 3).

“This was a big difference [from other classes], but it wasn't that hard to get used to because you get to actually come to class, relax, and look at each topic on its own and see what you have to do to get better information” (Student 3, FG2).

In addition to student comments, teachers/coaches also described many ways in which they noticed improvements in student engagement, achievement and efficacy in mathematics/ science in both years of the study. Table 10 (below) identifies the frequency of teacher/coach perceptions of student improvements identified in the interviews conducted each year.

**Table 10**

*Teacher/Coach Perceptions of Student Improvement.*

Themes in Year 1 Interviews	Frequency	Themes in Year 2 Interviews	Frequency
Improved engagement	6	Improved engagement	3
Opportunities for success for students who may not traditionally have experienced success	5	Improved collaborative/problem solving culture	3
Increased confidence and risk-taking demonstrated	4	Students feel more successful	2
Improvements in social and/or collaborative skills	3	Improved awareness of own learning	2
Improved resilience/stick-with-it-ness/struggle	3	Program growth	1
Evidence of academic improvement	2	Culture of problem solving	1
Improved ability to show work/use mathematical notation	2	Dropping failure rates	1
Students traditionally demonstrating memorization/mimicking improved problem-solving skills	2		
Improved language (EAL) opportunities and skills	1		
Increased enrolment in high level courses (e.g. physics, advanced calculus)	1		

## Discussion

As previously outlined, the BTC Initiative contained many elements of effective PD as outlined in the literature. What is interesting in the data is the variability of growth that occurred for the teachers interviewed. According to Clarke and Hollingsworth (2002),

The context in which teachers work (the Change Environment) can have a substantial impact on their professional growth. The school context can impinge on a teacher's professional growth at every stage of the professional development process: access to opportunities for professional development; restriction or support for particular types of participation; encouragement or discouragement to experiment with new teaching techniques; and administrative restrictions or support in the long-term application of new ideas. (p. 962)

The impact of classroom/school/division contexts was evident in the interview data of several participants. For example, Participant 5 cited competing initiatives and changes in colleagues as negatively influencing their ability to make significant changes in practice. Participant 1 also noted a lack of colleagues to collaborate with in their school, tempering their ability to engage in robust growth. Further, Participant 7 elected to use Liljedahl's strategies only for enrichment purposes. This was done largely due to the way they and their colleague divided up the students in their classes (they worked with the stronger group). In these ways (and as illustrated in Figure 2), classroom context impacted both the *Domain of Practice* and the *Domain of Consequence*. Similarly, school and divisional context influenced both the *External Domain* (stimulating growth) and the *Domain of Practice* (when teachers made choices about whether or how to engage with the strategies shared).

In addition to the influence of context on teacher change, personal circumstances also profoundly impacted teachers' ability to engage in growth networks and sustained changes in practice (through the *Personal Domain*). The personal time teachers had available to work on changes in practice and their commitment to the PD and implementing new pedagogical strategies were significant factors. Those teachers who had significant personal time and were committed to the process (i.e. Participants 6 and 10) were able to engage in robust growth and change. Those who did not experienced decreased impacts on practice.

In terms of the BTC Initiative and the school division involved in the case study, these findings suggest that several factors should be considered when recruiting teachers in their Critical Friends initiatives. First, the classroom environment of teachers, including their student makeup, should be considered. For participants in the study, additional sessions on topics such as problematic behaviours, attending to diversity in learning styles, and addressing background holes due to the COVID-19 pandemic would have been helpful. Moreover, attention should be paid to ways of fostering opportunities for collaboration within schools (such as recruiting teachers in pairs/teams), and ways of avoiding competing initiatives. Finally, the personal commitment of teachers to making changes in practice and the amount of personal time they have to engage in such change are of critical importance. Interviewing teachers prior to having them sign up to assess these factors could be helpful, as could clear expectations about engagement, ongoing support and encouragement. Moreover, finding ways to increase time provided for teachers to engage in the work of change (e.g. follow-up sessions or additional time to plan and create) could decrease the burden on the personal time of teachers, making PD more accessible.

In terms of what is known about effective teacher PD, the findings from the study highlight the importance of personal and contextual factors in supporting and sustaining changes in teacher practice. While the BTC Initiative checked many of the boxes in terms of what is known about effective teacher PD; and although the initiative resulted in significant changes in practice for some teachers and positive perceptions of student engagement, achievement and efficacy; the success of the BTC project for individual teachers came down to the contexts in which they worked, their personal commitment to the initiative, and the time they had available to engage in the difficult work of change.

### **Limitations and Future Directions**

While the findings of this case study are limited due to the nature of the case study, making it impossible to extrapolate findings to PD in other contexts, they do call into question what can be done to strengthen the positive (and minimize the negative) influences of personal and contextual factors on teacher change. Others providing teacher PD that checks many (or all) of the boxes in terms of what is known about effective teacher PD may want to pay close attention to these factors to help support teachers, thereby optimizing individual teacher change. Future research in the areas of teacher motivation and decision-making around changing practice may be helpful to understand how best to support teachers. Moreover, further analysis of the personal and professional time spent engaging in changes to practice, as well as effective strategies for carving out more time in the professional lives of teachers, could also be helpful in both improving the effectiveness of PD initiatives and the individual success of teachers making robust and sustained changes in practice.

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