

A Rationale for the Junior-Senior Secondary Mathematics Curriculum 2.0

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Abstract

This paper proposes a rationale that supports a renewal of our predominantly 19th century curriculum for Grades 7–12, identified as Mathematics 1.0. It was originally established in the mid 1800s to prepare learners mostly from upper-class families to succeed in a post-industrial society. Today's digital revolution has changed society remarkably, and the variety of learners has certainly broadened, but Mathematics 1.0 fundamentally remains the same Plato-based (Platonist) curriculum due to its social-political power, which is documented in the article. The major changes to society's culture and the composition of learners have caused faults in Mathematics 1.0 (e.g., a relevance deficit). For the majority of learners, school mathematics has mostly become an obsolete, inequitable, and harmful rite-of-passage into adulthood, to varying degrees. A renewed curriculum, Mathematics 2.0, is rationalized and specific suggestions are offered. The minority of learners who successfully pursue mathematics to varying degrees would experience small changes in their new Mathematics 1.2.

Keywords: school mathematics, humanistic, curriculum differentiation, relevance



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In a professional AP/AOL poll conducted by Ipsos (2005) involving 1,000 adults (ages 20 to 39 years), 23% picked mathematics as their “*favourite*” subject, while 37% “*hated*” mathematics. The margin of error was ± 3.1 . Public polls are not academic research, but they can alert us to a potentially serious problem that needs researching. In this case, it would seem that junior-senior (Grades 7–12) secondary mathematics is inadequately serving about 37% of learners—a crisis situation (Borovik, 2017). Oesterle (2018) concurred:

For experienced teachers, it can be difficult to alter the way we have been teaching, especially if it seems to “work” for at least some significant portion of our students [e.g., 23%]. But generations of students who become adults who “hate math” and [the] pervasive avoidance of mathematics in North America, suggest that what we have been doing is not “working” (p. 161).

With each generation, this negativity towards mathematics gets recycled when the 37% pass it along to their children, nieces, and nephews, for elementary teachers to handle.

Oesterle is not alone. Skovsmose and Greer (2012) concluded, “For too many people, their experience of school mathematics is personally, emotionally, and intellectually dehumanizing. It does not have to be like that” (p. 383). This dehumanizing contributes substantially to the crisis surrounding mathematics (Skovsmose, 2019). What we have been doing is certainly not working, and it is not the teachers’ fault. Both the content relevance gap between the curriculum content and what learners experience in their lives (Aikenhead, 2021a) must share the blame along with the “historical trend of teaching more math to more students at younger ages” (Furr, 1996, p. 8).

There is extensive evidence that few fundamental changes have occurred to the content and philosophy of school mathematics for Grades 7 to 12 since the inception of public schools about 170 years ago, when learners were taught “arithmetic, algebra, and geometry” (Nikolakaki, 2016; Willoughby, 1967, p. 4) from a curriculum established during the post-industrial revolution of the Victorian era. This first curriculum, designated “Mathematics 1.0” in this article, was taught only to upper-class learners (Andrews, 2016) because “arithmetic was too difficult for the lower class” (Nikolakaki, 2016, p. 277). Learner characteristics have changed dramatically since then, but not the curriculum’s Plato-based (Platonist) philosophy. Consequently, “Mathematics 1.0” also refers to our present-day curriculum, while “Mathematics 2.0” is the proposed revised curriculum for the majority of learners. “Mathematics 1.2” is a slightly revised curriculum for the minority. Some of these learners will enrol in both pathways due to the everyday relevance of Mathematics 2.0.

The purpose of this article is three-fold and focuses on the majority of learners’ abilities to (a) identify major faults associated with Mathematics 1.0, (b) document the social-political power that sustains Mathematics 1.0 in schools today for all learners, and (c) provide ideas for Mathematics 2.0 based on the mathematics education literature and inferred from Mathematics 1.0’s major faults. Mathematics 1.2 is not given much attention due to space limitations.

Learner Diversity

The first major faults associated with Mathematics 1.0 are that it lacks sufficient attention to learner diversity by collapsing it into dichotomies and it stereotypes learners with expectations

that they all could be highly proficient, rather than viewing them in terms of reaching their individual potentials.

The politics of simplistic dichotomies (e.g., STEM—science, technology, engineering, and mathematics—verses non-STEM learners) is avoided here in order to create a continuum that captures nuances in learners’ diversities, based on their self-identities with respect to mathematics (Heyd-Metzuyanim, 2017; Nasir, 2002; Nasir et al., 2008). The depth of learning is a function of the degree to which a learner’s mathematical self-identity is forged or enhanced (Thomas & Berry, 2019). Darragh and Radovic (2018) defined self-identities as:

A socially produced way of being, as enacted and recognized in relation to learning mathematics. It involves stories, discourses, actions, decisions, and affiliations that people use to construct who they are in relation to mathematics, but also in interaction with multiple other simultaneously lived identities. (para. 1)

Ruef (2020) wrote, “To be successful in mathematics classes, students must negotiate and navigate the normative identity of the class—what counts as being ‘good at math’” (p. 22). A self-concept is an amalgam of many traits, predilections, interests and so forth, of which the dichotomies “like/dislike” and “good/poor” blend into the amalgam.

One of those traits, “learners’ values,” collaborates with the self-identity-based diversity of learners. In her research into people’s typical clusters of values, psychologist Gilligan (1982, as cited in Ernest, 2018,) distinguished between the sciences and humanities. Mathematicians tended to valorize a cluster of “*separated values* [emphasis added]: rules, abstraction, objectification, impersonality, unfeelingness, dispassionate reason and analysis” (p. 194); as opposed to a humanistic stance of living by valuing a cluster of “*connected values* [emphasis added]: relationships, connections, empathy, caring, feelings and intuition” (p. 195).

Aikenhead (2021a) developed a six-category continuum according to the proportions and degrees to which Canadian high school graduates’ self-identities harmonize with the mathematical self-identities of mathematicians (see Table 1). The categories range from learners who find mathematics disturbingly difficult to varying degrees (i.e., math-phobic, math-shy, math-disinterested learners). The math-phobic often develop psychological or physiological anxieties when forced to think mathematically, especially when being assessed (Ernest, 2018; Maloney et al., 2017).¹ On the other side of the continuum are the math-interested, math-curious, and math-oriented learners,² who see the world to varying degrees like mathematicians do—“through the lens of mathematics” (Ernest, 2019, p. 2).

Table 1
A Student Diversity Continuum of Mathematical Self-Identities

math-phobic (20%)	math-shy (24%)	math-disinterested (26%)	math-interested (20%)	a	b
70%			30%		

a=math-curious (6%)
b=math-oriented (4%)

Note. This table demonstrates a student diversity continuum showing the proportion of Canadian high school graduates’ mathematical self-identities, based on a synthesis of three major research studies (approximately to scale). Not for student streaming. (Based on Aikenhead, 2021b, who reported Saskatchewan figures.)

These six categories must be treated flexibly and tentatively because learners' categorization depends on many changeable factors (e.g., “the teacher, topic, grade level, classroom environment, degrees of past success, season,” Meyer & Aikenhead, 2021a, p. 104).

The categories are not for streaming learners, but have been proposed for discussing learner diversity more realistically. For example, the 37% of the young adults who hated mathematics (Ipsos, 2005) were likely math-phobic and math-shy learners. I do not make distinctions between the categories in this article. A detailed account of the calculations for Table 1 is found in Meyer and Aikenhead (2021a). Percent data were derived from proportionately integrating the PISA³ 2018 proficiency data (OECD, 2019, Table I.6.1, p. 105), transformed to Grade 12 graduates according to Frederick's (1991) data, and slightly adjusted to fit Card and Payne's (2017) Ontario 70/30 % figures.

The data create a skewed distribution of learners in favour of the majority who would likely avoid high school mathematics if they could (Holm & Kajander, 2012). Mathematics educators talk about developing a growth mindset (e.g., Boaler, 2013). This implies that a learner moves towards the right in Table 1. It does not necessarily mean that a learner becomes a math-oriented or math-curious, even though the youcubed-at-Stanford-University (2021) website claims: “Everyone can learn to the highest levels” (p. 1)—an instance of Platonists stereotyping learners.

A more nuanced understanding of learners' interests, potentials, and predispositions is required for maximizing an up-to-date, equitable, and successful mathematics education for Grades 7–12; that is, Mathematics 2.0.

OECD's PISA Assessment Project: A Validity Audit

The OECD is an economic body in the highly political field of international investments. Its educational system turns out to be far more political than educational, a rather natural alignment with Platonist mathematics.

Every 3 years, ministries of education, the public media, and organizations such as the Fraser Institute give major attention to learners' mathematics, science, and reading scores reported in the PISA results, a project comprised of questionnaires and tests. This public attention reveals the first invalid aspect of PISA—cherry picking the results by the Ministries and the media by attending to only the learners' scores.

Cherry Picking the Results

The OECD judges the quality of an educational jurisdiction by the following three major criteria that the PISA project investigates: (a) test scores, statistically normalized and given greatest attention; (b) equity, highly valued by Canada's multicultural self-image and related to equity's influence on test scores by a learners' socio-economic status and immigrant identity; and (c) frugality, related to a government's expenditure on education compared to its learners' mean scores. How valid is a judgement concerning a province's or a country's education system if two out of the three major variables are all but ignored? Wikipedia calls this “cherry picking” (2021) the results, meaning, “suppressing evidence.” It invalidates an educational judgement. However, for a political judgement, the issue is changed to: Who is blamed for poor test results?—the teachers or the government? The answer is obvious.

When a three-variable combined analysis is actually conducted (Craw, 2015), it makes an enormous difference to the countries' PISA ratings (Aikenhead, 2017): For example, when

“Finland, Estonia, and Canada [were ranked] at 11, 12, and 13 (respectively) on the basis of student performance alone. ...the three-factor analysis resulted in Finland, Estonia, and Canada being tied for the *top* [number 1] PISA ranking” (p. 125). Cherry picking destroys validity. PISA does not calculate a combined analysis, as Craw (2015) did.

Sampling Invalidity

PISA is essentially a very complex polling project. Data are collected from randomly selected schools within an educational jurisdiction. With these data, inferences are made for the whole educational jurisdiction. Therefore, the ensuing statistics’ veracity depends on the sampling validity. However, some schools manipulate their student sample (Sjøberg & Jenkins, 2020, p. 7). For these jurisdictions, PISA is a high-stakes test related to attracting economic investments to their country. They are known to have given low achieving students the day off away from the testing cite. “PISA even claims that low educational performance on its assessment has an economic impact on [underdeveloped countries]” (Sriraman, 2017, p. xi).

At the same time, “PISA is a ‘low-stakes’ test,” (Sjøberg & Jenkins, 2020, p. 8) in more affluent countries where some learners have a tendency to not take it seriously. This decreases both their test scores and the test’s validity to some degree, in terms of comparing test scores among provinces or countries. Randomness is undermined and consequently the ensuing statistics produce faulty results.

Invalidities by Associations

Correlations (associations) are not to be identified with causations, but they can signal a potential serious problem for a measurement’s validity. Parallel to mathematics scores, high PISA scores are associated with a “strongly negative orientation towards science” (Sjøberg & Jenkins, 2020, pp. 3–4). The negative “side-effects” (p. 4) can easily arise from the enormous pressure on schools to succeed at the highest level. This tends to cause serious physiological and psychological harm to math-phobic and math-shy learners (Ernest, 2018, 2019; Zhao, 2017). Math-curious or math-interested could become math-disinterested or math-shy learners in such a classroom environment. Such negative effects were also evident in learners’ “very low self-confidence and self-efficacy related to science and mathematics” (Sjøberg & Jenkins, 2020, p. 4). Countries with the highest test scores “were at the very bottom of the ranking of students’ interest” in the subject (Bybee & McCrae, 2011). Simply put, high PISA scores may be harbingers of bad news for learners. Sjøberg and Jenkins (2020) stated, “the PISA science scores correlate negatively with Future science orientation ($r = -0.83$)” (p. 3).

Content and Writing Style Invalidities

The OECD (2013) claims to assess mathematics literacy: “It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens” (p. 23). Yet this relevant content is not part of the actual PISA test. It was not assessed. This would seem to be either fallacious posturing or a serious flaw in the test’s content validity.

What the test seems to assess is the Platonist content chosen by a European-based group of experts with some international consultations. What is the match between that content and someone’s local curriculum content? This factor seriously reduces the content validity of PISA scores:

The available evidence shows that culture may play a more significant role than pedagogy in determining the educational achievements of country, a finding that should be of great concern to anyone with an interest in improving both mathematics teaching and student achievement. (Andrews, 2016, p. 19)

Thus, when comparing one's own province to others, a major compromise to the PISA validity is not being able to control for a major variable: culture. Yet, a senior fellow at the Fraser Institute (Allison, 2021) called PISA "The gold standard for comparing school and student performances around the world" (para. 5)—perhaps an economist who needs to learn more about education assessment.

A measure of the PISA's test's educational validity is the degrees to which the test content and the cultural ways of posing mathematical questions correspond to the curriculum content and cultural context of the learners writing the test. For instance, Québec's culture is much closer to a European culture than Canada's Anglophone provinces are, which might explain Québec's PISA mathematics scores being consistently and significantly higher than all other provinces (Andrews, 2016). This causes PISA's validity to be highly problematic with respect to its content for writing style.

Challenges to Test Validity by a Test Items' Context

The culture of the PISA test developers can affect its validity. For those who are trapped in a Platonist decontextualized test item mode (i.e., a context foreign in varying degrees to math-phobic, shy, and disinterested learners—the 70% group), learners will respond differently compared to a developer who authentically contextualizes the test item in an everyday context familiar to the learners taking the test. Hypothetical contexts similar to a conventional word problem in textbooks are generally irrelevant or non-authentic to the 70% group to varying degrees (Serder & Jakobsson, 2015).

Devlin (2005) compared the arithmetic success rate when children, working as Third World street-stall sellers, solved the "same" arithmetic calculations in three different contexts: (a) in action at their stall, (b) typical textbook formatted word problems in their own language, and (c) decontextualized arithmetic questions. The average scores were 98%, 74%, and 37%, respectively. This is a rather special situation, but it affords insight into the difference that relevant contexts can have on measures of mathematical proficiencies. Procedural mathematics in a workplace differs qualitatively from pure and applied mathematics (Aikenhead, 2021b).

Le Hebel et al. (2017) statistically analyzed the individual PISA items' proficiency with respect to it distinguishing between learners who tended toward high and low achievements on their total PISA test scores. The researchers discovered that PISA test items generally work well among high achievers (i.e., the math-oriented, math-curious and perhaps the math-interested) but not among the low achievers. This suggests that PISA tests should be given to a province's or country's high mathematics achievers in order to increase the validity of individual items, and hence, the entire test.

Conclusion to This Validity Audit

If the OECD targeted the group that will most likely impact a country's GDP, it should focus on the 30% of the proficient mathematics high school learners. Thus, the sampling would be limited to the top mathematical 30% to 35% of a school's 15-year-olds. This would mitigate

only some of PISA's current crisis validity problems. In spite of this, however, it certainly has political power. Canada's Fraser Institute lauds it (Allison, 2021).

Sjøberg and Jenkins (2020) concluded, "PISA scores and rankings are not facts, nor are they objective or neutral outcomes of the project. There is therefore an important task facing the science [and mathematics] education community, namely to give the PISA project the rigorous scholarly examination it deserves" (p. 11).

A plethora of evidence supports the conclusion that PISA is essentially "a political project masquerading as an educational tool" (Aikenhead, 2017, p. 145). When a testing system is educationally broken due to serious problems with its validity (i.e., "validity" generally defined as its trustworthiness at measuring what it claims to measure), is it ethical to continue to use it? The present validity audit would conclude it is not.

The First Public School Curriculum

The source of major faults associated today with Mathematics 1.0 lies in the dynamics crafted by elite university mathematicians when public schools were established during the 19th century. The circumstances of this historical event are mentioned here in order to contrast them with today's more critical historical circumstances: changing our current profit society into a sustainable society, Carney,(2020b) stated: "We've been trading off the planet against profit, living for today and leaving it to others to pay tomorrow" (para. 12). In addition, "Society is beginning to place greater value on sustainability" (para. 5).⁴

In Late Medieval times, the mathematics of commerce became very popular in the schools, all of which were privately owned by entrepreneur instructors (Devlin, 2002). Academic mathematics developed in the fledgling universities in Europe (Daileader, 2020). Both the commercial schools and academic mathematicians supported the Industrial Revolution (late 18th to early 19th centuries). This historical event needed a public school system with a mathematics curriculum to educate workers for the new industries and associated businesses, all encouraged by an expanding open-market philosophy of a profit society at the time.

Ernest (2019) wrote, "Up to the 19th century, the classics (Latin and Greek) were used to occupy the same symbolic role" (p. 5) as mathematics occupies today: a screening device, under the guise of "rigorous standards," to keep fewer learners from graduating from high school and, "A hoop to jump through just to prove you could" (Russell, 2017, p. 25). Ernest (2019) said, "As mathematics professionals and insiders, we are complicit in this over-valuing of mathematics" (p. 6).

Such was the case about 170 years ago. At public debates, the nature of the high school curriculum was hotly contested. On one side were educators and businessmen who wanted practical everyday mathematics to comprise the curriculum, while on the other side were the elite Platonist professors dedicated to Plato's absolutist philosophy (Nikolakaki, 2016) along with some of the general public wishing to emulate the long established private Latin Schools (Willoughby, 1967).

Spurious tactics adopted by the professors are detailed in Aikenhead (2017). For instance, the Platonists vehemently claimed that mathematics was value-free, despite a public repository of mathematics values and ideologies at the time. Ernest (1991) pointed out, "The values of the absolutists [were] smuggled into mathematics, either consciously or unconsciously, through the *definition of the field* [emphasis added]" (p. 259). The Platonists drew on an ancient binary,

logical versus irrational, in order to invent their own theoretical binary: “formal mathematical discourse” versus “informal mathematical discourse” (p. 53). They arbitrarily assigned school mathematics to the formal discourse category that followed Plato’s absolutist philosophy of mathematics. The informal discourse category contained all of the features that made mathematics a human endeavour; for example, its commerce and political–societal contexts (Skovsmose, 2016), including its ideologies and values by which it operates (Ernest, 2016a, 2016b). Informal discourses were denigrated by the Platonists as not meeting the academic standards of Plato’s purity-of-the-mind axioms. In the end, the elite professors won the debate, and Platonist Mathematics 1.0 became school mathematics.

Pais (2012) pointed out the effect on today’s mathematics teaching and research caused by the Platonists’ hidden agenda to silence any talk beyond Plato’s purity-of-the-mind mathematics:

This concealment is essential to maintain the role of school as an ideological state apparatus. Seeing school as a place free of ideology disables bringing ideological struggles to school. All enterprises undertaken by teachers to unmask the “invisible” ideology are immediately accused of being ideological acts. In this way, the dominant ideology ensures that no ideology is present in school except, of course, the dominant one. The dominant one is precisely the one that presents itself as ideologically free. (p. 70)

This was how public secondary school mathematics began about 170 years ago: as an ideological screening device and as a cultural rite-of-passage out of high school for all learners.

The Platonists’ arbitrary invention of the mathematical formal and informal discourse categories has ensured that no fundamental innovation in secondary mathematics would take place if it conveyed the idea that the knowledge of mathematics is a human, pluralist, cultural endeavour guided by values and ideologies—the essence of Mathematics 2.0.

On the one hand, its Victorian era relevance enhances the 30 % of graduates with the credentials to pursue a STEM related profession. On the one hand as mentioned above, a curriculum deemed relevant by Victorian era mathematicians is experienced as a negative rite-of-passage for many of the 70 % (Ernest, 2018, 2019). As a result, the 19th century, elite mathematicians have been able to reach into our 21st century’s digital revolution society and decide what is appropriately relevant for all mathematics learners today.

Professional Influences

In addition to the powerful political forces promoting Platonist mathematics for all learners, two international professional groups influence Canadian mathematics education: the Organization for Economic Cooperation and Development (OECD), and the USA’s National Council of Teachers of Mathematics (NCTM). They all subscribe to “a *fallacious argument* [emphasis added] that because of the great utility and power [of mathematics to the needs of society, therefore,] all students must be taught and certified in mathematics to the highest possible level” (Ernest, 2019, p. 4).

Pais (2012) also criticized “the ideological injunction that you really need mathematics to attain [full] citizenship” (p. 65). This untruth tends to be repeated many times by teachers justifying their subject to their learners, many of whom know, however, the claim is an exaggeration: For example, “Really? Think about people you know. Aren’t there many who do

not have a solid grounding...in mathematics that are living full and productive lives? Isn't it offensive to tell such people that they are dysfunctional?" (Greer & Mukhopadhyay, 2012, pp. 239–240).

The OECD (2013) provided the following advice: “An understanding of mathematics is central to a young person’s preparedness for life in modern society. ...Mathematics is a critical tool for young people as they *confront issues and challenges* [emphasis added] in personal, occupational, societal, and scientific aspects of their lives” (p. 25). As documented above, this rhetoric is contradicted by a validity audit of the OECD’s prize project, PISA.

In their book, *Catalyzing Change*, the NCTM (2018) proposed: “Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics” (p. 37). NCTM (2018) defined “mathematically demanding courses as those courses that...maintain the integrity of the mathematical standards” (p. 84). These standards replicate Platonist mathematics.

Largely, these quotations target all learners with similar expectations of meeting “learning standards.” This stereotypes learners subtly and ignores their need to be treated as a highly diverse group (in spite of such pronouncements within the quoted documents). The two organizations seem to target the 30% of Grade 12 graduates. Their rhetoric tacitly vacillates between theoretically all learners and realistically the 30% minority of the learners.

By doing so, they have, by and large, implicitly shifted the valid mathematical needs of the 30% minority onto the 70% majority, thereby causing a much higher failure rate in the 70% majority of the learners—even in “foundational” mathematics high school pathways. In what way is that ethical?

My reading of the OECD and NCTM documents discerns a preoccupation with pure or applied mathematics (i.e., Platonist “Essential Concepts”) that meets high standards measured by standardized test results. This puts the perennial onus on teachers to improve their pedagogy, even though it is much more likely that the Platonist curriculum content is in crisis due to its relevance gap. The OECD and NCTM are mainly silent on societal, historic, and philosophical issues that adults confront in relation to mathematics authentically contextualized in the everyday world. This silence ensures that the curriculum remains overloaded with Platonist mathematics (formal mathematical discourses), leaving no room in the curriculum for adding innovative content related to mathematics’ interactions with, and influences on, individual citizens and society (informal mathematical discourse) (Duchscherer et al., 2019; Meyer & Aikenhead, 2021b).

Enculturated Power Politics of Platonist Mathematics

Platonist Mathematics 1.0’s original authority was politically established by its “formal discourse” designation in a theoretical dichotomy. Since then, it has been able to wield massive socio-politico-economic power globally. The continued use of a validity-flawed Platonist PISA test is only a symptom of Platonist mathematics’ societal power. The present section explores a deeper cause, thereby clarifying causes for these faults in Mathematics 1.0: the indoctrination or enculturation of citizens.

As mentioned above, Carney (2020a) described the historical transformation that Western societies underwent in the 18th century, from being ruled by values to being societies ruled by financial profit for the purpose of national progress, popularized by Adam Smith’s (1776) open-

market theory introduced in his book *Wealth of Nations*. Platonist mathematics played a central role in this transformation (Nikolakaki, 2016). This increased Platonist mathematics' use value considerably, thereby forging a strong relationship between it and the field of economics.

In such a society predicated by profit, “Numbers were considered to be objective...and they were more convincing than opinions or rhetoric. The ability of arithmetic not only to produce progress but also measure it, explains why it held this [privileged] position in the educational system” (Nikolakaki, 2016, p. 279). Platonist mathematics became entrenched in Eurocentric nations' progress, which added to its high status. For instance, it “can operate as a political pacifier by making controversial readings and handlings appear neutral and objective” (Skovsmose, 2019, p 1).

Western cultures' global market economies exercise power pervasively over the investment world, the military-industrial complex, democratic governments, and education (Carney, 2020a). Aikenhead (2017) pointed out:

Many potential culture-based innovations to school mathematics have been sacrificed on the altar of Platonist mathematical content. Platonist content deserves serious attention, to be sure. But its current excessive emphasis has created (a) myths, (b) beliefs in those myths, (c) social power bestowed upon those beliefs, and (d) privileges gained by that social power. Unless broken, this cycle will repeat itself for generations to come. (p. 119)

This cycle maintains the inequitable “social class structures in society” (Jorgensen, 2016, p. 127), and notably the continued colonization of Canada's Indigenous peoples (Aikenhead, 2017).

School mathematics generally serves as a gatekeeper that often illogically limits the future for many high school graduates. Ernest (2019) wrote, “The assessment system in mathematics provides a social obstacle and a filter fabricating a reduction in life chances” (p. 4). Ernest (2019) made an important distinction: “In modern society the *exchange value* of mathematics far outweighs its *use value* [emphasis added]” (p. 5). An exchange value refers to the socio-political power and privileges gained by scoring high marks and taking more mathematics classes. A use value refers to the degree to which the curriculum content is actually used by most adults.

The inequities caused by Mathematics 1.0 function as a filter for social advancement are made possible by the following:

- The very high value that the elite in society holds for old wives' tales about mathematics in general, such as one's intelligence being indicated mostly by mathematics achievement;
- School mathematics' status over other subjects allows mathematics professors and teachers to avoid justifying their curriculum content and to ignore taking responsibility for its negative ethical consequences, such as with the 37% of young adults hating what they were offered at school; and
- The spurious way mathematics educators established their original 19th century public-school curriculum, Mathematics 1.0, in order to deflect any future fundamental changes being made to it. (Its spurious nature was explained in this article's section: The First Public School Curriculum.)

To sustain its high status, mathematicians issue periodic, half-true, propaganda statements; for example, “The supremacist position maintained by many mathematician educators who regard abstract mathematics as the crowning achievement of the human intellect, and school mathematics as the transmission of its products” (Mukhopadhyay & Greer, 2012, p. 860). As a result, politicians do not seem to take responsibility for the negative ethical consequences due to the social inequities resulting from school mathematics (Andrews, 2016, Ernest, 2018) and the harmful sophisticated algorithms created by mathematicians (O’Neil, 2017).

Simply put, the evidence suggests that much of the social power wielded by Platonist mathematics is born out of myths created and propagated by those who desire economic and social control over others living in a profit society (Carney, 2020a). These myths have been so ensconced in Canadian culture that once any child or learner is enculturated into mainstream culture it will take a major crisis to challenge the myths. One such crisis is upon us. Carney (2020b) noted that another worldwide societal transformation is underway: from a profit society to a sustainable society, this time for the purpose of raw human survival. You cannot make a profit if there is no inhabitable planet. We need a parallel fundamental transformation in Grades 7–12 mathematics curricula for provincial and national leaders and their informed citizens who will be involved in shepherding the transition to a sustainable society. Mathematics 2.0 will harmonize with such a need. Mathematics 1.0 will only continue to retard the transformation.

Humanistic Implications for Mathematics 2.0

Mathematics 2.0 is intended for the math-phobic, math-shy, and math-disinterested learners currently studying the pure and applied Platonist mathematics, also known as “in-school mathematics” (Aikenhead, 2021b, pp. 29–30). It is “the antithesis of human activity—mechanical, detached, emotionless, value-free, and morally neutral” (Fyhn et al., 2011, p. 186). Thus, “*out-of-school* mathematics” (Aikenhead, 2021b, pp. 30-31) has been found to shift learners’ categorization in Table 1 toward the right (Aikenhead, 2017; Barta et al., 2014; Boaler, 2015). Out-of-school mathematics is a human endeavour because it is based on humans producing, applying, or using mathematics. This will have rigorous academic standards, but they will be appropriate for people who do not see the world so much as a mathematician does. By incorporating a teaching context drawn from a humanities or cross-cultural perspective, there will be academic standards related to a mathematics literacy (Aikenhead 2021a, 2021c; Barta et al, 2014) that Platonists write about in their propaganda.

Interestingly, the Ipsos (2005) poll recorded the respondents’ favourite school subjects (margin of error of ± 3.1). Mathematics was 31% compared to the humanities combination (English, History, Social Studies, Arts, and Religion/Philosophy) of 69%. These figures are a confirmation of Card and Payne’s (2017) 30% and 70% results, and a clear indication that the humanities would make an excellent context for learning out-of-school mathematics (Aikenhead, 2021c).

Humanistic mathematics (e.g., Ravn & Skovsmose, 2019; Sriraman, 2017) has promise for being one of three themes for Mathematics 2.0. Aikenhead (2021c) and Hall (2021) exemplified the first them by demonstrating what a historical theme in humanistic mathematics looks like. A second theme is “mainstream culture-based mathematics” (Aikenhead, 2021a), which entails the following three aspects of mathematical interactions in adult life:

- Math-in-use in their workplace, community, home, and personal lives, which become contexts for learning mathematics (Barta et al., 2014);
- Platonist math-in-action, which in society refers to mathematics providing the foundation for the technological, medical, industrial, military, economic, and political systems, along with the ethical interrogation of the makers of mathematics, for example, O’Neil’s (2017) *Weapons of Math Destruction*, taboo content in *Mathematics 1.0*; and
- Culture and nature of Platonist mathematics, for example, being aware of its axiomatic logical thinking that deduced its false façade that claimed to be value-free, non-ideological, non-cultural, purely objective in its use, and certain in its answers, whereas Platonist mathematics has been shown to be value-laden (e.g., truth, rationalism, universalism objectivism, beauty, ethics, purity, certainty, and objectivism), ideological (e.g., quantificational, universalism, objectivism, foundationalism, and rationalism), and cultural (Larvor, 2016): “As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality” (Einstein, 1921).

A third theme for countries with Indigenous citizens is enhancing mathematics with Indigenous mathematizing (Aikenhead, 2017; Meyer & Aikenhead, 2021b; Nicol et al., 2013).

This triad of themes for *Mathematics 2.0* affords unlimited connections to Saskatchewan’s four curriculum goals: understanding mathematics as a human endeavour, number sense, logical thinking, and spatial sense (Saskatchewan Ministry of Education, 2008, pp. 7–9).

Because of its pluralistic orientation, *Mathematics 2.0* will avoid indoctrinating learners into an absolutist philosophy of mathematics. Thus, it will present Platonism in the context of learning how Canadian culture functions with Platonist mathematics. *Mathematics 2.0* will tend to avoid hypothetical contextualizations and applications of pure mathematics because both of these widen the relevance gap.

Mathematician Borovik (2017) identified a crisis regarding the credibility of *Mathematics 1.0*’s content and philosophy in the 21st century. In his chapter, he also pointed out a much different crisis for the 23% who favoured mathematics over all other school subjects (Ipsos, 2005). Some of this group’s future as professional mathematicians “will be filling an increasingly small number of jobs, which really require mathematical competence (I call them *mathematical makers*)” (p. 309)—the Silicon Valley crew, for example. Everyone else will be “*end users* of technology saturated by mathematics—which however, will remain invisible to them” (Ipsos, 2005, p. 309).

A group of users also work in areas of education, science, technology, engineering, economics, and architecture. Borovik’s (2017) prediction that end users in business and industry will need less advanced mathematics has already occurred: for instance, in the banking, investment and airline industries (Hoyle et al., 2001) and in engineering (Edwards, 2010): “The vast majority of scientists, engineers and actuaries only use Excel and eighth grade level mathematics” (p. 19).

For the majority 70% of Grade 12 graduates could focus on “basic numeracy and awareness” (Borovik, 2017, p. 309) of out-of-school mathematics contextualized by the following:

- How people actually use mathematics (Aikenhead, 2021a): “Mathematics becomes best understood by *how it is used*” (Barta et al., 2014, p. 3);
- Analyzing mathematics-related social issues, for which mathematics has offered helpful solutions or has caused an issue (Ernest, 1991; Ernest et al., 2016); and
- Some aspects of the humanities, discussed above.

Today, teachers are forced to work against a growing relevance gap between what learners experience as mathematics in their everyday world, compared to what textbooks try to get learners to superimpose on their world. The relevance gap becomes wider and wider as our digital revolution evolves at exponential rates (Borovik, 2017). If curriculum developers continue to ignore the digital revolution, then both the Ipsos (2005) 37% crisis of adults who “hate” mathematics will grow, and mathematics educators’ concerns will worsen.

Fundamental Innovations Already Initiated

What is the Saskatchewan Ministry of Education’s (2008) position concerning a humanistic mathematics? One of its four goals is understanding “mathematic as a human endeavour” (p. 9). This certainly invites Saskatchewan mathematics educators to organize innovative projects to move in these culture-based or humanistic directions for Mathematics 2.0.

Three recent projects illustrate what can be accomplished. First, Indigenous “Culture-Based School Mathematics for Reconciliation and Professional Development” (Meyer & Aikenhead, 2021a, 2021b) mentored non-Indigenous rural mathematics teachers to learn a few Indigenous mathematizing activities and then develop a lesson plan that taught Indigenous perspectives to learners as they engaged in these activities. According to Duchschere et al. (2019), “When a teacher makes a clear connection between the Indigenous mathematizing and an analogous idea in the Western mathematics curriculum, this Western content is generally introduced to students who are already motivated to learn” (p. 4).

Secondly, Saskatchewan’s Provincial Education Sector (2021), a consortium of Ministry of Education personnel with 28 Boards of Education, recently justified the integration of Indigenous ways of knowing throughout their website “SaskMATH” based on the curriculum’s goal: “Mathematics as a human endeavour.” Their project organized about 45 chosen mathematics teachers, school division consultants, Division Directors, and mathematics education researchers to develop a website of evidence-based best practices for teaching number sense in Grades 1 to 12. It integrates Indigenous ways of knowing wherever possible throughout the website.

Thirdly, an emerging project creating six teaching modules, “Culture-Based School Mathematics” (Aikenhead, 2021a, p. 26), was initiated but its classroom research delayed due to COVID-19. At the appropriate age level of learners, each module addresses: math-in-use (i.e., procedural mathematics actually used by employers and employees), math-in-action (i.e., the economic, social and political interactions of mathematics in a community or with society), the culture of Platonist mathematics (e.g., its foundational beliefs, values, ideologies, history, type of reasoning, valid arguments, relationship to music and games, etc.; often called the nature of mathematics), and Indigenous perspectives (e.g., involving Indigenous mathematizing, reconciliation, etc.). All the above includes some contexts for learning analogous conventional curriculum content.

Some may say a culture-based or humanistic renewal of Saskatchewan's school mathematics is not feasible for a majority of teachers and learners in Grades 7–12. An honest reply is that it has already been achieved, for the most part, in Saskatchewan's science program for Grades 1–11 (Aikenhead & Elliot, 2010). If Saskatchewan's school science program can do it, why not Saskatchewan's school mathematics?

Conclusion: Mathematics 2.0 and 1.2

In her chapter, "Morality and Mathematics," Muntersbjorn (2016) asked the fundamental question that underscores this article:

Why should students be required to take mathematics courses [emphasis added]? If so few pupils have a taste or talent for mathematics, why are we obligated to teach mathematics to as many of the next generation as possible? Universal mathematical literacy seems a noble (if naïve) goal of any culture...But whence the normative force behind the claim, "all pupils must learn mathematics"? (p. 387)

This article suggests answers to those probing questions by considering the complexity of the faults in Platonist mathematics education—faults that come with a history embedded in its culture while denying it is cultural. Platonist mathematics interacts with a diversity of learners with its ideologies of universalism and quantification while denying it is ideological. It champions an absolutist philosophy, which it defends with nefariously creative dichotomies (e.g., formal and informal discourses) dressed up in inductive reasoning.

The poet T. S. Eliot (1934) captured the essence of Mathematics 1.0's faults when he wrote, "Where is the wisdom we have lost in knowledge?" (line 15). Platonist mathematics became a hegemonic force acquiring social-political power (Bishop, 1990; Greer & Mukhopadhyay, 2016) with its sophisticated intellectual knowledge, defined similarly in every dictionary. A definition of "mathematics" that best represents Mathematics 2.0 is the version distilled from Bishop's (1988)'s humanistic perspective: "Mathematics is a symbolic technology for building relationships between humans and their social and physical environments" (p. 147). This approach is open to a curriculum's inclusion of wise decision making.

Living in the digital revolution, faced with the crisis of climate change, humanity needs a mathematics education that embraces both knowledge and wisdom. Mathematics 2.0 is a prime candidate. Thus, ministries of education need to develop both Mathematics 2.0 for the 70% math-phobic, math-shy, and math-disinterested learners and an updated Platonist mathematics (i.e., Mathematics 1.2) for the 30% math-interested, math-curious, and math-oriented, with a degree of humanistic enrichment balanced off by removing an equal amount of obsolete content from Mathematics 1.0.

I conclude by discussing some directions to take so Mathematics 2.0 becomes a reality. By understanding the full spectrum of learners' mathematical self-identities and their value clusters (i.e., degrees of separated and connected clusters; Table 1), educators can now improve Grades 7–8 and Grades 9–12 mathematics by the following factors:⁵

- Coordinating the diversity of learners in terms of the degree to which their worldviews, self-identities, and values converge with a mathematician's, from highly discordant to highly harmonious;

- Coordinating the diversity of appropriately relevant mathematics content for learners to come to know from highly humanistic or culture-based school mathematics (identified above as math-in-use, math-in-action, Indigenous mathematizing, and the nature of Platonist mathematics), which targets the 70% of learners to mostly Platonist mathematics and the 30 % of learners to enrichment in Grades 7 to 9 (e.g., math clubs), and precalculus courses enhanced with Mathematics 2.0-like projects in Grades 10–12.
- Supporting teacher professional development programs and/or research projects, province wide, to develop a few prototype teaching units for each of the six grades to generate new units every second year over 10 years.
- Always addressing the elimination of teachers’ learned or indoctrinated blind spots, such as not being able to distinguish between out-of-school mathematics and applied mathematics (Aikenhead, 2021b). These blind spots can marginalize Saskatchewan’s “mathematics as a human endeavour” goal as well as marginalizing math-disinterested, math-shy, and math-phobic learners, thereby adding to Saskatchewan’s social inequities (Jorgensen, 2016).
- Maintaining the curriculum’s present three-pathway structure, with an eye to:
 - a. Slightly updating The Workplace & Apprenticeship’s pathway to meet the realities of their math-in-use as advised by on-sight workers, and exploring examples of Indigenous mathematizing, math-in-action, and the nature of mathematics related to their workplace.
 - b. Transforming the Foundation of Mathematics 20 and 30 pathway into something like The Culture of Mathematics in the Real World 10, 20, and 30; giving the goal “mathematics as a human endeavour” greater emphasis while highlighting math-in-use; and deleting from the curriculum an equal amount of “dispensable mathematical baggage from the 19th and 20th centuries”—a description that those who have escaped their indoctrination into Platonism might use.
 - c. Slightly modifying Precalculus 10, 20 and 30 by deleting content obsolete in today’s digital age, including what is no longer applicable to mathematics, science, architecture, technology, engineering, and medicine, etc.; adopting a few International Baccalaureate topics; and replacing some current content with a few enrichment projects per course related to mathematics as a human endeavour; topics chosen individually or by small groups of collaborating learners.
- Giving special attention to learners in Grade 9 mathematics. Get them to recollect their Grades 7–9 mathematics and its implications for their choice of pathways for Grades 10–12. Use some precalculus situations throughout the course as elective material to add to learners’ database for their decision. Let learners choose whether their elective material assessment counts toward their final mark.
- Making learner transfers between the pathways explicitly feasible and highly individualized. Monitor each semester on the social and ancestral equity of the pathways and then implement ways to augment equity, such as providing free tutoring for those in need. The current term “intervention” is fueled by the separated values cluster (Gilligan, 1982) that tend to undermine equity agendas.

- Setting aside the notion that Saskatchewan learners must be assessed by written tests that are based largely on a (a) European type of mathematics curriculum, (b) European genre of composing test questions (e.g., PISA mathematics scores in French Québec verses the rest of Canada's), and (c) the fallacy that those test scores represent the quality of the educational jurisdiction (Andrews, 2016; Sjøberg, 2015; Sjøberg & Jenkins, 2020).
- Beginning negotiations with postsecondary institutions.

Ministries of education should embrace the international economic-based movement to sustainability (Carney, 2020b) as part of renewing its Grades 7–12 mathematics program effectively and urgently. For best results, they should follow the “smart money.” Our profit society is slowly metamorphizing into a sustainable society (Carney, 2020b). “As climate risks will ultimately affect every sector of the economy...We won't have a financial system if we don't have a planet” (p. 5). Mark Carney captured today's learners' immediate future. It is an urgent crisis.

¹ “The key concern here is to alleviate boredom and drudgery for mathphobes and those who suffer from math anxiety” (Greer, 2012, p. 115).

² In the spirit of full disclosure, I am a math-oriented person.

³ The PISA (Program for International Student Assessment project “is owned and governed by member states in the Organization for Economic Cooperation and Development (OECD)” (Sjøberg & Jenkins, 2020, p. 1).

⁴ Mark Carney is the former governor of the Bank of Canada and then the Bank of England.

⁵ Saskatchewan is typical of many provinces, so it is used here as an example.

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